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ABSTRACT

This paper uncovers the mechanism and assumptions underlying how population growth induces structural transformation. We construct two-sector models that give analytically tractable closed-form solutions. If sectoral goods are consumption complements, population growth induces a more than proportionate relative price rise compared to the relative marginal product of labor drop in a sector with stronger diminishing returns to labor, and shifts production factors towards that sector.

Our work points to a two-stage development process: (1) in early development, population growth shifts production factors to agriculture; and (2) when agricultural productivity growth is fast enough, production factors move out of agriculture.

Keywords: Structural transformation; Population growth effect; Relative price effects; Relative marginal product effects

JEL Codes: E1, N1, O5

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“[P]opulation increases, and the demand for corn raises its price relatively to other things—more capital is profitably employed on agriculture, and continues to flow towards it”.
(David Ricardo 1821, 361)

1 INTRODUCTION

The concepts of population growth and structural transformation are vital to the study and practice of economic development. At least since Malthus (1826), who argued that population multiplies geometrically and food arithmetically to raise food prices and depress real wages, scholars have been exploring the links between population growth and economic development (Kuznets 1960; Boserup 1965; Simon 1977; Kremer 1993; Diamond 2005). Recently, Leukhina and Turnovsky (2016) brought forward the idea that population growth induces structural transformation.¹ Their focus was on simulating the contribution of population growth to structural development in England. However, the mechanism by which population growth induces structural transformation was not adequately addressed in their paper. The central thesis of this paper is to further delineate this mechanism, by constructing two-sector models that give analytically tractable closed-form solutions of structural development.

Traditionally, economists have focused on structural transformation away from agriculture since industrialization in the Western world (Clark 1960, 510-520; Kuznets 1966, 106-107; Chenery and Syrquin 1975, 48-50). Seldom has attention been paid to the sectoral shift towards agriculture before the industrialization breakthrough (see the English and United States examples in sections 3 and 7), when income, technology and capital stock progressed slowly. Indeed, population growth was perhaps the most salient change in the Malthusian economies, that contributed to structural transformation in pre-industrial times.²

We construct two models to explain the two-stage development process implied above. Our models are simple enough to deliver closed-form solutions that track the mechanisms and crucial assumptions by which population growth, as well as technological progress and capital deepening, induces structural transformation.

The basic model (section 4) examines structural transformation in pre-industrial times. It is

¹ Structural transformation refers to factor reallocation across different sectors in the economy. More broadly, Chenery (1988, 197) defined structural transformation as “changes in economic structure that typically accompany growth during a given period or within a particular set of countries”. He considered industrialization, agricultural transformation, migration and urbanization as examples of structural transformation.

² The role of population growth on structural transformation is often overlooked. One exception is Johnston and Kilby (1975, 83-84), who stated that population growth determines the rate and direction of structural transformation. They defined the rate of structural transformation from agriculture to non-agriculture as $RST = \frac{L_n}{L_t}(L'_n - L'_t)$, where L_n is non-farm employment, L_t is total labor force, and L'_n and L'_t are their respective rates of change. They noted that, ceteris paribus,

“[t]he impact of a high rate of population growth (L'_t) is, of course, to diminish the value of ($L'_n - L'_t$). In Ceylon, Egypt, and Indonesia, high rates of population growth equalled or surpassed L'_n in recent decades so that structural transformation ceased or was reversed.”

a two-sector (agricultural and manufacturing), two-factor (labor and land) model.³ In the model, the representative household views agricultural and manufacturing goods as consumption complements, while agricultural production possesses stronger diminishing returns to labor. Holding sectoral factor shares constant, population growth will increase manufacturing output relative to agricultural output, raising the relative price of agricultural goods (relative price effect). At the same time, the increase in labor input in the two sectors will reduce the relative marginal product in the agricultural sector (relative marginal product effect). Given that the two sectoral goods are consumption complements, the relative price effect originating from the households' unwillingness to consume too few agricultural goods relative to manufacturing goods will outpace the relative marginal product effect. Since factor return equals output price times marginal product, this will relatively boost agricultural factor returns and draw production factors towards the agricultural sector. We call this the *population growth effect on structural transformation*.⁴ We will apply this model to simulate the rise (and fall) of agricultural labor share in pre-industrial England (AD1521-AD1745). Note that as the focus of this paper leans more towards the theoretical side, the simulations are more for illustrative purposes.

Next, the unified model (section 5) examines structural transformation in the modern times. Population is still an important component. We extend the basic model by allowing for technological progress and including capital as another production input. There are four relative price effects that foster structural transformation in the model, namely *the agricultural technology growth effect, the manufacturing technology growth effect, the population growth effect* and *the capital deepening effect*. From our analytical solution, to move production factors away from agriculture, we need a fast enough agricultural technology growth rate so that the agricultural technology growth effect overrides the other three relative price effects. We will apply this model to simulate the fall of agricultural factor shares in the modern United States (AD1980-AD2100). Part of the success of our work is the reconciliation of the fall in agricultural land share throughout development, which is not featured elsewhere in the structural transformation literature.

The next section reviews the relevant literature. Section 3 describes historical facts related to sectoral shifts in pre-industrial England and the modern United States. Section 4 develops the basic model. Section 5 extends it to the unified model. In section 6 we calibrate the two models to simulate sectoral shifts in pre-industrial England and the modern United States respectively. Section 7 highlights some discussion. Section 8 concludes.

2 RELATED LITERATURE

Our work is related to three bodies of literature. The first is the causes of structural transformation, which can be traced back to the work by Harris and Todaro (1970). They hypothesized that when the rural wage is lower than the expected urban wage, labor will migrate

³ In this paper, “manufacturing sector” refers to non-agricultural sector.

⁴ David Ricardo mentioned that population growth attracts capital towards the agricultural sector through the relative price effect. See his quote ahead of the Introduction.

from the rural to the urban sector. In their model labor movement is a disequilibrium phenomenon in the sense that unemployment exists. The literature has evolved to consider how structural transformation occurs within frameworks where full employment and allocation efficiency are achieved. Income effect and relative price effect originating from technology growth have become standard channels to explain structural transformation within these frameworks. The former is a demand-side approach, which assumes a non-homothetic household utility function, usually with a lower income elasticity on agricultural goods than on non-agricultural goods. Hence income growth throughout development process will shift demand away from the agricultural goods, fostering a relative agricultural decline in the economy. For example, Matsuyama (1992), Laitner (2000), Kongsamut et al. (2001), Gollin et al. (2002, 2007), Foellmi and Zweimüller (2008), Gollin and Rogerson (2014) shared this property. The latter is a supply-side approach, which emphasizes that differential productivity growth across sectors will bring along relative price changes among consumption goods. And the resulting direction of sectoral shift will depend on the degree of substitutability among different consumption goods. For example, Hansen and Prescott (2002), Doepke (2004), Ngai and Pissarides (2007, 2008), Acemoglu and Guerrieri (2008), Bar and Leukhina (2010) and Lagerlöf (2010) shared this feature. Acemoglu and Guerrieri (2008) proposed capital deepening as an additional cause that generates structural transformation through the relative price effect.

In the recent years, the literature has evolved to look into alternative explanations for structural transformation. For example, models with education/training costs (Caselli and Coleman 2001), tax changes (Rogerson 2008), barriers to labor reallocation and adoption of modern agricultural inputs (Restuccia et al. 2008), transportation improvement (Herrendorf et al. 2012), scale economies (Buera and Kaboski 2012a), human capital (Buera and Kaboski 2012b) and international trade (Uy et al. 2013) have been proposed. See Herrendorf et al. (2014) for a survey. Leukhina and Turnovsky (2016) posited population growth as another cause of structural transformation. They relied on simulating FOC conditions from a general equilibrium model to study structural development. In comparison, this paper will derive analytical closed-form solutions for sectoral share evolution, which shed light on the underlying mechanism and crucial assumptions of the population growth effect on structural transformation (sections 4.2 and 5.4).⁵

The second set of literature is related to developing unified models for structural transformation. Echevarria (1997), Acemoglu and Guerrieri (2008), Dennis and Iscan (2009), Duarte and Restuccia (2010), Alvarez-Cuadrado and Poschke (2011), and Guilló et al. (2011)'s works were in this direction. They constructed micro-founded models by blending at least two of the following causes of structural transformation: non-homothetic preference, biased technological progress and capital deepening. They either employed the models to simulate cross-sectional or time-evolving sectoral share patterns, or evaluated the relative importance of the above causes in accounting for historical structural changes. Hansen and Prescott (2002), Leukhina and Turnovsky (2016) also constructed unified models, where population growth is a cause of

⁵ Population growth is exogenous in this paper (sections 4 and 5). This allows us to focus on how population growth by itself gives rise to structural transformation. See Ho (2016) who incorporates the population growth effect on structural transformation in a framework with endogenous population growth to reconcile Eurasian economic history.

structural transformation. Again, as mentioned in the previous paragraph, they shared the methodology of relying on FOC simulations but not closed-form solutions to analyze structural development.

The third body of literature is related to the effect of population growth on per capita income evolution in growth models. In Solow (1956), Cass (1965) and Koopmans (1965)’s exogenous growth models, diminishing marginal product of capital assures saving in the economy just to replenish capital depreciation and population growth in the steady state. A change in population growth rate has just a level effect but no growth effect on per capita income evolution in the long run. In the AD1990s, Jones (1995), Kortum (1997) and Segerstrom (1998) proposed semi-endogenous growth models, which incorporate R&D and assume diminishing returns to R&D. In steady states, these models predict that per capita income (or real wage) growth rate increases linearly with population growth rate.⁶ To summarize, the above literature predicts a non-negative effect of population growth rate on per capita income growth rate in steady states. In contrast, in our growth models with land as a fixed production factor, faster population growth can adversely affect per capita income growth rate, even when the economies have attained their asymptotic growth paths (sections 4.3 and 5.5).

3 HISTORICAL EVIDENCE

This section documents historical evidence related to structural transformation between agricultural and manufacturing sectors in pre-industrial England (section 3.1) and the modern United States (section 3.2). Besides motivating our models in sections 4 and 5, these historical evidence will also be used for calibrations in section 6.

3.1 *Structural Transformation in pre-industrial England*

Sectoral shift occurred in pre-industrial England. Figure 1 depicts Clark (2010, 2013)’s estimates of agricultural labor share in England during AD1381-AD1755.⁷ Agricultural labor share gradually rose during the early Modern Period and decisively declined after the

⁶ In the early AD1990s, before the emergence of semi-endogenous growth models, Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) built first-generation endogenous growth models. They predict that economies with larger population sizes (rather than population growth rates) would grow faster (population scale effect). The reasons are that those economies could employ more research scientists and there are larger markets for successful innovative firms to capture.

However, the empirical evidence did not support the population scale effect (Jones 1995). In the late AD1990s, Peretto (1998), Howitt (1999) and Young (1998) constructed Schumpeterian endogenous growth models. These models nullify the population scale effect by introducing endogenous product proliferation: as population increases, it attracts entry of new product varieties. This reduces effectiveness of productivity improvement as R&D resources are spread thinner across the expanding research frontier, and reward to product quality innovation is dissipated.

⁷ In this paper, the term “agricultural labor share” refers to the proportion of labor allocated to the agricultural sector, but *not* the fraction of national income labor captures. Similar interpretation holds for the terms “agricultural capital share”, “agricultural land share”, “manufacturing labor share”, “manufacturing capital share” and “manufacturing land share”.

mid-seventeenth century.⁸

INSERT FIGURE 1 HERE

Structural transformation is commonly known to be caused by income growth (Kongsamut et al. 2001), biased technological progress (Ngai and Pissarides 2007) and capital deepening (Acemoglu and Guerrieri 2008). Before the Industrial Revolution, Britain was in its Malthusian era when income stagnation and slow capital accumulation characterized the country's development. We also assume there was neglectable manufacturing technological progress in this period. Hence only agricultural productivity growth is left to explain sectoral shift. Table 1 shows Clark (2002)'s estimates of annual agricultural productivity growth rate in England during AD1525-AD1795. The magnitude of agricultural productivity growth during AD1525-AD1745 was quite moderate by modern standards.

INSERT TABLE 1 HERE

There is indeed another potential candidate which contributes to structural transformation: population growth. Figure 2 depicts Mitchell (1988) and Pamuk (2007)'s population estimates in England during AD1400-AD1801. Since AD1400, the English population had stayed at roughly 3 million for more than a century. It then rose at rates comparable to modern standards up till around AD1660. After that it stagnated at about 5 million until the eve of the Industrial Revolution.

INSERT FIGURE 2 HERE

We hypothesize that the interplay of population growth effect and agricultural technology growth effect on structural transformation explains agricultural labor share movement in pre-industrial England. In section 4 we will abstract technological progress and construct the basic model. This allows us to focus on the population growth effect on structural transformation. Agricultural productivity growth will be added in section 6.1 when we simulate sectoral shift in pre-industrial England.

3.2 *Structural Transformation in the modern United States*

Sectoral shift has also occurred in the modern United States.⁹ Figure 3 depicts U.S. Bureau of Economic Analysis, or BEA (2016), and World Bank (2016)'s estimates of agricultural capital share (solid line), labor share (dashed line) and land share (dotted line) in the United States throughout AD1947-AD2013. All these factor shares were generally declining during their respective time frames.

INSERT FIGURE 3 HERE

We hypothesize that, in the modern times, agricultural and manufacturing technological

⁸ Broadberry et al. (2013) also provided estimates of agricultural labor share in England during AD1381-AD1861. Their estimates showed qualitatively the same rise-and-fall trend as Clark (2010, 2013)'s one, but the turning point occurred earlier, during the mid-sixteenth century. We will stay with Clark (2010, 2013)'s estimates throughout this paper.

⁹ For the United States, the term "agricultural sector" refers to the agricultural, forestry, fishing and hunting sectors defined by U.S. Bureau of Economic Analysis, or BEA (2016) in their NIPA Tables. The term "manufacturing sector" refers to all sectors other than agricultural, forestry, fishing and hunting.

progresses, population growth and capital deepening explain structural transformation. We examine the evolution of related variables in the United States during the late-twentieth and early-twenty-first centuries. Figure 4 depicts the farm total factor productivity in the United States during AD1948-AD2011, provided by U.S. Department of Agriculture (2016). Agricultural productivity was in general rising, and its growth had accelerated since the AD1980s.

INSERT FIGURE 4 HERE

Figure 5 depicts the annual multifactor productivity (SIC measures) for private nonfarm business sector in the United States during AD1948-AD2002, provided by U.S. Bureau of Labor Statistics, or BLS (2016). We use it to proxy manufacturing productivity. Manufacturing productivity was generally improving over time. It had suffered from a productivity growth slowdown since the AD1980s.

INSERT FIGURE 5 HERE

Figures 6 and 7 depict the number of full-time and part-time employees in the United States during AD1969-AD2013 and chain-type quantity indexes for net stock of fixed assets and consumer durable goods in the United States during AD1948-AD2013, provided by BEA (2016). Population growth and capital accumulation were both at work.

INSERT FIGURE 6 HERE

INSERT FIGURE 7 HERE

In section 5 we will construct a unified model to account for structural transformation through the interplay of population growth effect, technology growth effects and capital deepening effect. In section 6.2 we will calibrate the unified model to simulate sectoral shift in the modern United States.

4 THE BASIC MODEL

4.1 Model setup (two-sector, two-factor)

We set up the basic model to examine the population growth effect on structural transformation. Households are homogenous. There are two sectors (agricultural and manufacturing) and two production factors (labor and land) in the economy. Markets are complete and competitive. Factors are mobile across the two sectors. Time is continuous and indexed by t .

The population at time t , L_t , equals L_0 times e^{nt} , where L_0 is the initial population and n is the population growth rate. Each household is endowed with one unit of labor which is supplied inelastically. We assume households are altruistic towards their future generations. The representative household possesses lifetime utility function in the form of:

$$(1) \quad \int_0^\infty e^{-(\rho-n)t} \frac{\tilde{c}_t^{1-\theta}-1}{1-\theta} dt ,$$

where ρ is the discount rate, θ is the inverse of elasticity of intertemporal substitution, \tilde{c}_t is per capita consumption composite at time t .

The representative household makes consumption decisions $\{\tilde{c}_t\}_{t=0}^\infty$ subject to budget

constraints at $t \in [0, \infty)$. At time t , the household owns one unit of labor and $\frac{T}{L_t}$ unit of land.

By supplying them to the market, the household obtains a wage income of $W_t(1)$ and a land rental income of $\Omega_t \frac{T}{L_t}$, where W_t and Ω_t are the nominal wage rate and land rental rate at time t .¹⁰

Formally, the budget constraint facing the representative household at time t is:

$$(2) \quad \tilde{c}_t = \frac{W_t}{P_t}(1) + \frac{\Omega_t T}{P_t L_t} ,$$

where P_t is the consumption composite price at time t .

Per capita consumption composite is a constant elasticity of substitution (CES) aggregator of per capita purchase of agricultural and manufacturing goods:

$$(3) \quad \tilde{c}_t = \left(\omega_A \tilde{y}_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M \tilde{y}_{Mt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \omega_A, \omega_M \in (0,1), \quad \omega_A + \omega_M = 1, \quad \varepsilon \in [0, \infty),$$

where $\tilde{y}_{At} \equiv \frac{Y_{At}}{L_t}$ and $\tilde{y}_{Mt} \equiv \frac{Y_{Mt}}{L_t}$ are per capita purchase of agricultural and manufacturing goods

at time t respectively, ω_A and ω_M are measures of relative strengths of demand for the two sectoral goods, ε is elasticity of substitution between the two sectoral goods. We denote the two sectoral goods to be consumption complements if $\varepsilon < 1$, and to be consumption substitutes if $\varepsilon > 1$.

Agricultural goods, Y_{At} , and manufacturing goods, Y_{Mt} , are produced competitively according to Cobb-Douglas technologies, using labor and land as inputs:

$$(4) \quad Y_{At} = A_t L_{At}^{\alpha_A} T_{At}^{\gamma_A}, \quad \alpha_A, \gamma_A \in (0,1), \quad \alpha_A + \gamma_A = 1, \quad g_A \equiv \frac{A_t}{A_t} = 0 ,$$

$$(5) \quad Y_{Mt} = M_t L_{Mt}^{\alpha_M} T_{Mt}^{\gamma_M}, \quad \alpha_M, \gamma_M \in (0,1), \quad \alpha_M + \gamma_M = 1, \quad g_M \equiv \frac{M_t}{M_t} = 0 ,$$

where L_{At} and L_{Mt} , T_{At} and T_{Mt} are labor and land employed by the two sectors at time t ; A_t and M_t are agricultural and manufacturing productivities at time t ; α_A and α_M , γ_A and γ_M are labor intensities and land intensities in the two production sectors. In this section, to single out the population growth effect on structural transformation, we assume $A_t = A$ and $M_t = M$ for all t , that is, there are no technological progresses in the two sectors. Note that α_A and α_M measure the degree of diminishing returns to labor in the two sectors: the greater the values of these parameters are, the weaker diminishing returns to labor are.

Factor market clearing implies that the sum of factor demands from the two sectors equals aggregate factor supplies at each time t :

$$(6) \quad L_{At} + L_{Mt} = L_t ,$$

$$(7) \quad T_{At} + T_{Mt} = T ,$$

where T is the amount of land in the economy, which is fixed in supply for all time t .

¹⁰ To be more precise, the representative household also makes decision on whether to supply production factors to the agricultural or manufacturing sector. In equilibrium, factor returns in the two sectors will be equalized ((13) and (14)). Therefore we do not make a distinction between wages or land rentals in the two sectors in the representative household's budget constraint (2).

Equations (1)-(7) describe our model economy. To proceed, we define Y_t as the unique final output being produced competitively in the economy, using agricultural and manufacturing goods as intermediate inputs.

$$(8) \quad Y_t = \left(\omega_A Y_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_{Mt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Technically, final output is an aggregator of agricultural and manufacturing output that represents the representative household's consumption composite preference.¹¹

We normalize the price of final output as the numéraire in the economy for all time t , that is:¹²

$$(9) \quad 1 \equiv (\omega_A^\varepsilon P_{At}^{1-\varepsilon} + \omega_M^\varepsilon P_{Mt}^{1-\varepsilon})^{\frac{1}{1-\varepsilon}},$$

where the associated prices of agricultural and manufacturing goods at time t , P_{At} and P_{Mt} , are respectively:

$$(10) \quad P_{At} = \omega_A \left(\frac{Y_t}{Y_{At}} \right)^{\frac{1}{\varepsilon}},$$

$$(11) \quad P_{Mt} = \omega_M \left(\frac{Y_t}{Y_{Mt}} \right)^{\frac{1}{\varepsilon}}.$$

Note that the consumption composite price always equals the final output price, that is, $P_t = 1$ for all t .

Also, equation (2) can be aggregated as:¹³

$$(12) \quad L_t \tilde{c}_t = Y_t,$$

which has the interpretation of an economy-wide resource constraint. Hence the competitive equilibrium problem (1)-(7) can be reframed as a social planner's problem of maximizing (1) subject to (4)-(12).¹⁴

Since capital is absent, the social planner's problem can be broken down into a sequence of intratemporal problems, that is, maximizing (8) subject to (4)-(7), (9)-(11) for each time point t . Solving the intratemporal problem is equivalent to solving for the entire dynamic path in this model. Competition and factor mobility implies wages W_t and land rentals Ω_t in the agricultural and manufacturing sectors are equalized:

$$(13) \quad W_t = \omega_A \alpha_A \left(\frac{Y_t}{Y_{At}} \right)^{\frac{1}{\varepsilon}} \frac{Y_{At}}{L_{At}} = \omega_M \alpha_M \left(\frac{Y_t}{Y_{Mt}} \right)^{\frac{1}{\varepsilon}} \frac{Y_{Mt}}{L_{Mt}},$$

$$(14) \quad \Omega_t = \omega_A \gamma_A \left(\frac{Y_t}{Y_{At}} \right)^{\frac{1}{\varepsilon}} \frac{Y_{At}}{T_{At}} = \omega_M \gamma_M \left(\frac{Y_t}{Y_{Mt}} \right)^{\frac{1}{\varepsilon}} \frac{Y_{Mt}}{T_{Mt}}.$$

By defining manufacturing labor share as $l_{Mt} \equiv \frac{L_{Mt}}{L_t}$ and manufacturing land share as

$\tau_{Mt} \equiv \frac{T_{Mt}}{T}$, equations (13)-(14) can be rewritten as:

¹¹ Technically, the final output (8) should combine with the implied economy-wide resource constraint (12) to give the representative household's consumption composite form (3).

¹² See Appendix 3A for the proof in a more general setting with capital accumulation.

¹³ See Appendix 3B for the proof.

¹⁴ This is an application of the Second Fundamental Theorem of Welfare Economics: given markets are complete and competitive, we can consider the problem faced by the social planner to solve for the growth path of the economy.

$$(15) \quad l_{Mt} = \left[1 + \frac{\omega_A \alpha_A}{\omega_M \alpha_M} \left(\frac{Y_{Mt}}{Y_{At}} \right)^{\frac{1-\varepsilon}{\varepsilon}} \right]^{-1},$$

$$(16) \quad \tau_{Mt} = \left[1 + \frac{\gamma_A \alpha_M}{\gamma_M \alpha_A} \frac{1-l_{Mt}}{l_{Mt}} \right]^{-1}.$$

Note that agricultural labor and land shares are $l_{At} = (1 - l_{Mt})$ and $\tau_{At} = (1 - \tau_{Mt})$ respectively.

4.2 Population growth effect on structural transformation

Population growth is the sole exogenous driving force across time in the basic model. Proposition 1 states how the manufacturing factor shares l_{Mt} and τ_{Mt} and relative sectoral output evolve when population increases over time. We will focus on the $\varepsilon < 1$ case.¹⁵

Proposition 1 (Population growth effect): In a competitive equilibrium,

$$(17) \quad \frac{l_{Mt}}{l_{Mt}} = \frac{(\alpha_M - \alpha_A)(1-l_{Mt})n}{\frac{\varepsilon}{\varepsilon-1} - [(\alpha_M - \alpha_A)(1-l_{Mt}) + (\gamma_M - \gamma_A)(1-\tau_{Mt}) + \alpha_A + \gamma_A]} \begin{cases} < 0 & \text{if } \varepsilon < 1 \text{ and } \alpha_M > \alpha_A \\ > 0 & \text{if } \varepsilon < 1 \text{ and } \alpha_M < \alpha_A \end{cases},$$

$$(18) \quad \frac{\tau_{Mt}}{\tau_{Mt}} = \left(\frac{1-\tau_{Mt}}{1-l_{Mt}} \right) \frac{l_{Mt}}{l_{Mt}}, \text{ which follows the same sign as in (17).}$$

$$(19) \quad \begin{cases} \frac{\dot{Y}_{Mt}}{Y_{Mt}} - \frac{\dot{Y}_{At}}{Y_{At}} > 0 & \text{if } \varepsilon < 1 \text{ and } \alpha_M > \alpha_A \\ \frac{\dot{Y}_{Mt}}{Y_{Mt}} - \frac{\dot{Y}_{At}}{Y_{At}} < 0 & \text{if } \varepsilon < 1 \text{ and } \alpha_M < \alpha_A \end{cases}.$$

Proof: See Appendix 1.

Equations (17)-(18) show the closed-form solutions of sectoral share evolution, which illustrates the population growth effect on structural transformation. From (17), when $\varepsilon < 1$, population growth pushes labor towards the sector characterized by stronger diminishing returns to labor. The mechanism that drives labor shift is population growth combined with different degrees of diminishing returns to labor in the two sectors: they create a relative price change in sectoral goods, which dominates the relative marginal product effect, leading to structural transformation. Combine (10), (11), take log and differentiate to get the *relative price effect*:

$$(20) \quad \left. \frac{\partial \ln \left(\frac{P_{Mt}}{P_{At}} \right)}{\partial \ln L_t} \right|_{\text{constant } l_{Mt}, \tau_{Mt}} = \frac{1}{\varepsilon} (\alpha_A - \alpha_M) \begin{cases} < 0 & \text{if } \alpha_M > \alpha_A \\ > 0 & \text{if } \alpha_M < \alpha_A \end{cases}.$$

Holding factor shares allocated to the two sectors constant, population growth will lead to a relative price drop in the sector characterized by weaker diminishing returns to labor. On the other hand, combining (4), (5), taking log and differentiating gives the *relative marginal product effect*:

$$(21) \quad \left. \frac{\partial \ln \left(\frac{MPL_{Mt}}{MPL_{At}} \right)}{\partial \ln L_t} \right|_{\text{constant } l_{Mt}, \tau_{Mt}} = (\alpha_M - \alpha_A) \begin{cases} > 0 & \text{if } \alpha_M > \alpha_A \\ < 0 & \text{if } \alpha_M < \alpha_A \end{cases},$$

where MPL_{At} and MPL_{Mt} are marginal products of labor in the two sectors. Marginal product of labor will rise relatively in the weaker diminishing returns sector. From (20)-(21), if $\varepsilon < 1$, when population increases, the aforementioned relative price drop in the weaker diminishing

¹⁵ Using the United States data from AD1870-AD2000, Buera and Kaboski (2009) calibrated the elasticity of substitution across sectoral goods, ε , to be 0.5. See section 7 for a discussion on the importance of the ε term in the structural transformation literature.

returns sector will be proportionately more than the rise in relative marginal product of labor in the same sector. Since wage equals sectoral price times marginal product of labor, wage will fall relatively in the weaker diminishing returns sector. This will induce labor to move out of the weaker diminishing returns sector, until the wage parity condition (13) is restored.¹⁶ Intuitively, we can also understand the population growth effect as follows: when the two sectoral goods are consumption complements, households do not want to consume too few of either one of them. When population grows, if sectoral labor shares stay constant, sectoral output grows slower in the sector with stronger diminishing returns to labor. Hence labor will shift to this sector to maximize the value of per capita consumption composite.

Since labor and land are complementary inputs during production of sectoral goods, land use also shifts in the same direction as labor. Corollary 1 reinforces our result:

Corollary 1 (Embrace the land): In the basic model, suppose there are two sectors producing consumption complements in the economy: one is labor-intensive and the other is land-intensive. In the absence of technological progress, population growth shifts production factors from the labor-intensive sector to the land-intensive sector (manufacturing-to-agricultural transformation in case of $\alpha_M > \alpha_A$).

Corollary 1 illuminates structural transformation in a Malthusian economy. Given agriculture is the land-intensive sector, in the Malthusian era when technology and capital stockpile slowly, population growth will push production factors towards agriculture. We believe this explains the rise in agricultural labor share or ruralization of an economy in the early stages of development (sections 6.1 and 7).

Proposition 1 also has implications on the pace of structural transformation, effect of scale economies and relative sectoral output growth. First, from (17) and (18), given $\varepsilon < 1$, a rise in population growth rate would accelerate factor reallocation.¹⁷ The reason is, from (20), that a faster population growth would generate a larger relative price effect (relative to the relative marginal product effect in (21)) and speed up structural transformation.

Second, whether an increase in scale economies of a sector affects the direction of factor reallocation depends on which sector gets the scale boost. In our model, we interpret α_A and α_M as measures of the scale advantages in agricultural and manufacturing production respectively. In the long run, land is fixed. In an economy with population growth, weaker diminishing returns to labor (higher α_A or higher α_M) would allow the sectors to produce more output in the long run. Without loss of generality, assume initially $\alpha_M > \alpha_A$. First, consider an increase in scale advantage of manufacturing production originating from a rise in α_M , from (17) sectoral shift towards agriculture continues. Next, consider an increase in scale advantage of agricultural production originating from a rise in α_A . From (17), if α_A increases to a level higher than α_M , then sectoral shift changes direction towards manufacturing. Otherwise the sectoral shift towards agriculture continues. Note from the above two cases that an increase in scale advantage of one sector will not bring along factor reallocation in favor of it. This result contrasts with Buera and Kaboski (2012a)'s proposition that an increase in scale advantage of a sector (market services in

¹⁶ Note (13) can be rewritten as $P_{At}MPL_{At} = P_{Mt}MPL_{Mt}$.

¹⁷ Note that a rise in population growth rate would not affect the direction of factor reallocation in the basic model.

their case) could yield a relative rise in labor time allocated to that sector.¹⁸

Third, from (19), over time population growth relatively promotes output growth in the sector characterized by weaker diminishing returns to labor. Population growth affects relative output growth in the two sectors through two channels: (1) sectoral production function channel: holding sectoral factor shares constant, this channel relatively promotes output growth in the sector with weaker diminishing returns to labor; (2) factor reallocation channel: given $\varepsilon < 1$, population growth pushes factors towards the sector with stronger diminishing returns to labor and relatively favors output growth in that sector. Overall, the first channel dominates.

4.3 Asymptotic growth path

We study the implication of population growth on the asymptotic growth path of the economy, which is summarized in proposition 2.

Proposition 2 (Asymptotic growth path): In the asymptotic growth path, denote $a^* \equiv \lim_{t \rightarrow \infty} a_t$, $g_a^* \equiv \lim_{t \rightarrow \infty} \left(\frac{\dot{a}_t}{a_t}\right)$, $y_t \equiv \frac{Y_t}{L_t}$ as per capita final output or per capita income in the economy, if $\varepsilon < 1$,¹⁹

$$\begin{aligned} & \begin{cases} l_M^* = 0 \text{ and } \tau_M^* = 0 & \text{if } \alpha_M > \alpha_A \\ l_M^* = 1 \text{ and } \tau_M^* = 1 & \text{if } \alpha_M < \alpha_A \end{cases}, \\ & g_{Y_A}^* = \alpha_A n, \quad g_{Y_M}^* = \alpha_M n, \quad g_Y^* = \min\{\alpha_A n, \alpha_M n\}, \\ & g_Y^* = \begin{cases} -(1 - \alpha_A)n, & \text{if } \alpha_M > \alpha_A \\ -(1 - \alpha_M)n, & \text{if } \alpha_M < \alpha_A \end{cases}. \end{aligned}$$

Proof: See Appendix 1.

Given $\varepsilon < 1$, in the asymptotic growth path, the sector with stronger diminishing returns to labor tend to draw away all labor and land in the economy. The rate of output growth in this sector will be slower than that in the other one. This sector will also determine the growth rate of final output. In our model, population growth puts a drag on per capita income growth rate even in the asymptotic growth path.²⁰ The higher the population growth rate is, the faster per capita income diminishes. This differs from the literature's prediction of a non-negative effect of population growth rate on per capita income growth rate in the steady states (section 2). The drag on per capita income growth rate originates from the presence of land as a fixed factor of sectoral production. Due to diminishing returns to labor, the limitation land puts on per capita income growth becomes more and more severe as population grows over time. The faster population grows, the quicker per capita income deteriorates due to this problem, and the larger is the resulting drag. Per capita income keeps on shrinking over time, and the economy ultimately ends up with stagnation.²¹

¹⁸ See proposition 6 in Buera and Kaboski (2012a)'s paper. Buera and Kaboski (2012a) measured scale advantage of a sector in terms of maximum output that a sector can produce due to the existence of capacity limit of intermediate goods. A sector with a larger capacity limit enjoys a greater scale advantage. In contrast, in our interpretation, a sector enjoys a scale advantage when it possesses weaker diminishing returns to labor.

¹⁹ In our closed-economy setting, per capita final output (y_t) equals per capita income ($\frac{W_t}{P_t} + \frac{\Omega_t}{P_t} \frac{T}{L_t}$).

²⁰ The population growth drag is the $-(1 - \min\{\alpha_A, \alpha_m\})n$ term.

²¹ Our basic model shares the Malthusian (1826)-Ricardian (1821) pessimism with respect to the

5 THE UNIFIED MODEL

5.1 Model setup (two-sector, three-factor)

We construct the unified model to examine how population growth, technological progress and capital accumulation affect structural transformation in the modern times. There are two sectors (agricultural and manufacturing) and three production factors (labor, capital and land). Technological progress occurs in both sectors. The crucial modeling feature that distinguishes from the literature is that we include land as a fixed production factor in all the two sectors. The motivation is that, land is an important input for the agricultural sector, and we observe declines in agricultural land share in contemporary high-income countries (see the United States example in Figure 3).²² Any theories aiming at explaining modern agricultural-to-manufacturing transformation should capture this fact.²³

Consider an economy which starts with L_0 identical households, and the population growth rate is n . Population at time t is:

$$(22) \quad L_t = L_0 e^{nt}.$$

Each household is endowed with one unit of labor, which is supplied inelastically. The representative household holds utility function in the form of:

$$(23) \quad \int_0^\infty e^{-(\rho-n)t} \frac{\tilde{c}_t^{1-\theta}-1}{1-\theta} dt,$$

where ρ is the discount rate, θ is the inverse of elasticity of intertemporal substitution, \tilde{c}_t is per capita consumption composite at time t .

The representative household makes his or her consumption decisions subject to budget constraints at $t \in [0, \infty)$:

$$(24) \quad \frac{\dot{K}_t}{L_t} = \frac{W_t}{P_t} (1) + r_t \frac{K_t}{L_t} + \frac{\Omega_t T}{P_t L_t} - \tilde{c}_t,$$

where $\frac{\dot{K}_t}{L_t}$ is the instantaneous change in per capita capital stock at time t , $\frac{K_t}{L_t}$ and $\frac{T}{L_t}$ are capital

and land each household owns at time t , $\frac{W_t}{P_t}$, $r_t \left(= \frac{R_t}{P_t} - \delta \right)$ and $\frac{\Omega_t}{P_t}$ are real wage rate, interest rate and land rental rate in terms of consumption composite price at time t . At each time t , the

ultimate agricultural stagnation. According to Ricardo (1821), in the absence of technological progress, with diminishing returns to land use, population growth will eventually drain up the entire agricultural surplus, cutting off the incentive for agricultural capitalists to accumulate fixed capital. The economy ends up with agricultural stagnation. Malthus (1826) pointed out that, as population multiplies geometrically and food arithmetically, population growth will eventually lead to falling wage (and rising food price), pressing the people to the subsistence level.

²² The World Bank (2016) provided estimates of agricultural land (% of land area) for the high-income countries, which declined from 38.6% in AD1961 to 30.0% in AD2013.

²³ Although Hansen and Prescott (2002), Leukhina and Turnovsky (2016) included land as a fixed production factor in their two-sector models, they only included land in one of the sectors (agriculture). Hence there will never be land allocated to the Solow/manufacturing sector in their models, making reconciliation of declines in agriculture land share impossible.

instantaneous change in per capita capital stock equals the sum of individual real wage, capital interest and land rental incomes, minus real individual spending on consumption composite.

Per capita consumption composite at time t is defined as:

$$(25) \quad \tilde{c}_t = \left(\omega_A \tilde{y}_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M \tilde{y}_{Mt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} - \frac{\dot{K}_t}{L_t} - \frac{\delta K_t}{L_t},$$

$$\omega_A, \omega_M \in (0,1), \omega_A + \omega_M = 1, \varepsilon \in [0, \infty),$$

where $\tilde{y}_{At} \equiv \frac{Y_{At}}{L_t}$ and $\tilde{y}_{Mt} \equiv \frac{Y_{Mt}}{L_t}$ are per capita purchase of agricultural and manufacturing goods

at time t respectively, ω_A and ω_M are the relative strengths of demand for the two sectoral goods respectively, and ε is elasticity of substitution between the two sectoral goods. Note that the representative household only values a portion of the CES aggregator of purchased sectoral goods, after investment and depreciation have been deducted from it, as the consumption composite.

Agricultural and manufacturing goods, Y_{At} and Y_{Mt} , are produced competitively according to Cobb-Douglas technologies, using labor, capital and land as inputs:

$$(26) \quad Y_{At} = A_t L_{At}^{\alpha_A} K_{At}^{\beta_A} T_{At}^{\gamma_A}, \quad \alpha_A, \beta_A, \gamma_A \in (0,1), \alpha_A + \beta_A + \gamma_A = 1, g_A \equiv \frac{\dot{A}_t}{A_t},$$

$$(27) \quad Y_{Mt} = M_t L_{Mt}^{\alpha_M} K_{Mt}^{\beta_M} T_{Mt}^{\gamma_M}, \quad \alpha_M, \beta_M, \gamma_M \in (0,1), \alpha_M + \beta_M + \gamma_M = 1, g_M \equiv \frac{\dot{M}_t}{M_t},$$

where L_{At} and L_{Mt} , K_{At} and K_{Mt} , T_{At} and T_{Mt} are labor, capital and land employed by the two sectors at time t ; α_A and α_M , β_A and β_M , γ_A and γ_M are labor intensities, capital intensities and land intensities in the two production sectors; A_t and M_t are agricultural and manufacturing productivities at time t , g_A and g_M are technology growth rates in the two sectors. Population growth and technological progresses are the exogenous driving forces across time in the unified model.

Factor market clearing implies that the sum of factor demands from the two sectors equals aggregate factor supplies at each time t :

$$(28) \quad L_{At} + L_{Mt} = L_t,$$

$$(29) \quad K_{At} + K_{Mt} = K_t,$$

$$(30) \quad T_{At} + T_{Mt} = T,$$

where T is the aggregate land supply in the economy, which is fixed over time.

Equations (22)-(30) describe our model economy. Markets are complete and competitive. Factors are freely mobile across sectors. By the Second Fundamental Theorem of Welfare Economics, we can reframe the decentralized problem of (22)-(30) as the problem faced by the social planner. We define Y_t as the unique final output at time t , which is produced competitively using agricultural and manufacturing goods as intermediate inputs:²⁴

$$(31) \quad Y_t = \left(\omega_A Y_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_{Mt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

²⁴ Similar to the previous section, final output is an aggregator of agricultural and manufacturing output that represents the representative household's consumption composite preference. Combining (31) and (32) yields (25). We think that (25) is the utility function implicitly embedded in Acemoglu and Guerrieri (2008)'s model.

We normalize the price of final output to one for all time points and (9)-(11), $P_t = 1$ for all t hold in this economy. Also, (24) can be aggregated to give an economy-wide resource constraint:²⁵

$$(32) \quad \dot{K}_t + \delta K_t + L_t \bar{c}_t = Y_t, \quad \delta \in [0, 1],$$

where δ is the capital depreciation rate, K_t is the level of capital stock at time t .

The social planner's problem is:

$$(33) \quad \max_{\{\bar{c}_t, K_t, L_{At}, L_{Mt}, K_{At}, K_{Mt}, T_{At}, T_{Mt}\}_{t=0}^{\infty}} \int_0^{\infty} e^{-(\rho-n)t} \frac{\bar{c}_t^{1-\theta}-1}{1-\theta} dt$$

subject to (9)-(11), (22), (26)-(32), given $K_0, L_0, T, A_0, M_0 > 0$.

The maximization problem (33) can be divided into two layers: the intertemporal and intratemporal allocation. In the intertemporal level, the social planner chooses paths of per capita consumption composite and aggregate capital stock over the entire time horizon $t \in [0, \infty)$. In the intratemporal level, the social planner divides the aggregate capital stock, total population and land between agricultural and manufacturing production to maximize final output at each time point t . We solve the problem starting from the lower level first, that is, the intratemporal level, and then move on to the higher intertemporal level.

5.2 Intratemporal level: Allocation between agricultural and manufacturing sectors

In the intratemporal level, at each time point t , the social planner maximizes the value of final output to allow him/her to choose among the largest possible choice set (32) in solving the intertemporal consumption-saving problem:

$$(34) \quad \max_{L_{At}, L_{Mt}, K_{At}, K_{Mt}, T_{At}, T_{Mt}} Y_t \quad \text{subject to (9)-(11), (26)-(31), given } K_t, L_t, T.$$

Competition and factor mobility implies that production efficiency is achieved. Wages W_t , capital rentals R_t and land rentals Ω_t are equalized across the agricultural and manufacturing sectors:

$$(35) \quad W_t = \omega_A \alpha_A \left(\frac{Y_t}{Y_{At}} \right)^{\frac{1}{\varepsilon}} \frac{Y_{At}}{L_{At}} = \omega_M \alpha_M \left(\frac{Y_t}{Y_{Mt}} \right)^{\frac{1}{\varepsilon}} \frac{Y_{Mt}}{L_{Mt}},$$

$$(36) \quad R_t = \omega_A \beta_A \left(\frac{Y_t}{Y_{At}} \right)^{\frac{1}{\varepsilon}} \frac{Y_{At}}{K_{At}} = \omega_M \beta_M \left(\frac{Y_t}{Y_{Mt}} \right)^{\frac{1}{\varepsilon}} \frac{Y_{Mt}}{K_{Mt}},$$

$$(37) \quad \Omega_t = \omega_A \gamma_A \left(\frac{Y_t}{Y_{At}} \right)^{\frac{1}{\varepsilon}} \frac{Y_{At}}{T_{At}} = \omega_M \gamma_M \left(\frac{Y_t}{Y_{Mt}} \right)^{\frac{1}{\varepsilon}} \frac{Y_{Mt}}{T_{Mt}}.$$

Defining the manufacturing labor, capital and land shares as $l_{Mt} \equiv \frac{L_{Mt}}{L_t}$, $k_{Mt} \equiv \frac{K_{Mt}}{K_t}$ and

$\tau_{Mt} \equiv \frac{T_{Mt}}{T}$ respectively, (35)-(37) can be rewritten as:

$$(38) \quad l_{Mt} = \left[1 + \frac{\omega_A \alpha_A}{\omega_M \alpha_M} \left(\frac{Y_{Mt}}{Y_{At}} \right)^{\frac{1-\varepsilon}{\varepsilon}} \right]^{-1},$$

²⁵ See Appendices 3A and 3C for the proof.

$$(39) \quad k_{Mt} = \left[1 + \frac{\alpha_M \beta_A}{\alpha_A \beta_M} \left(\frac{1-l_{Mt}}{l_{Mt}} \right) \right]^{-1},$$

$$(40) \quad \tau_{Mt} = \left[1 + \frac{\alpha_M \gamma_A}{\alpha_A \gamma_M} \left(\frac{1-l_{Mt}}{l_{Mt}} \right) \right]^{-1}.$$

Note that the agricultural labor, capital and land shares are $l_{At} = (1 - l_{Mt})$, $k_{At} = (1 - k_{Mt})$ and $\tau_{At} = (1 - \tau_{Mt})$ respectively. Equations (38)-(40) characterize the intratemporal equilibrium conditions.

Manipulating (38)-(40) and we obtain the following four propositions, which show how the sectoral shares l_{Mt} , k_{Mt} and τ_{Mt} respond to population growth, technological progresses and capital deepening:

Proposition 3 (Population growth effect): In a competitive equilibrium,

(41)

$$\frac{d \ln l_{Mt}}{d \ln L_t} = - \frac{(1-\varepsilon)(\alpha_M - \alpha_A)(1-l_{Mt})}{\varepsilon + (1-\varepsilon)[\alpha_M(1-l_{Mt}) + \alpha_A l_{Mt} + \beta_M(1-k_{Mt}) + \beta_A k_{Mt} + \gamma_M(1-\tau_{Mt}) + \gamma_A \tau_{Mt}]} < 0 \quad \text{if } \varepsilon < 1 \text{ and } \alpha_M > \alpha_A$$

$$> 0 \quad \text{if } \varepsilon < 1 \text{ and } \alpha_M < \alpha_A,$$

$$(42) \quad \frac{d \ln k_{Mt}}{d \ln L_t} = \frac{1-k_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln L_t},$$

$$(43) \quad \frac{d \ln \tau_{Mt}}{d \ln L_t} = \frac{1-\tau_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln L_t}.$$

Proof: See Appendix 1.

The mechanism for proposition 3 goes the same way as what we stated in section 4.2. *Ceteris paribus*, if $\varepsilon < 1$, population growth induces a more than proportionate relative price drop (compared to the relative marginal product of labor rise) in the sector characterized by weaker diminishing returns to labor. Labor shifts out this sector to maintain the wage parity condition (35). Since labor, capital and land are complementary sectoral inputs, they move in the same direction.

Proposition 4 (Agricultural technology growth effect): In a competitive equilibrium,

$$(44) \quad \frac{d \ln l_{Mt}}{d \ln A_t} = \frac{(1-\varepsilon)(1-l_{Mt})}{\varepsilon + (1-\varepsilon)[\alpha_M(1-l_{Mt}) + \alpha_A l_{Mt} + \beta_M(1-k_{Mt}) + \beta_A k_{Mt} + \gamma_M(1-\tau_{Mt}) + \gamma_A \tau_{Mt}]} > 0 \quad \text{if } \varepsilon < 1,$$

$$(45) \quad \frac{d \ln k_{Mt}}{d \ln A_t} = \frac{1-k_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln A_t},$$

$$(46) \quad \frac{d \ln \tau_{Mt}}{d \ln A_t} = \frac{1-\tau_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln A_t}.$$

Proof: See Appendix 1.

Proposition 5 (Manufacturing technology growth effect): In a competitive equilibrium,

$$(47) \quad \frac{d \ln l_{Mt}}{d \ln M_t} = - \frac{(1-\varepsilon)(1-l_{Mt})}{\varepsilon + (1-\varepsilon)[\alpha_M(1-l_{Mt}) + \alpha_A l_{Mt} + \beta_M(1-k_{Mt}) + \beta_A k_{Mt} + \gamma_M(1-\tau_{Mt}) + \gamma_A \tau_{Mt}]} < 0 \quad \text{if } \varepsilon < 1,$$

$$(48) \quad \frac{d \ln k_{Mt}}{d \ln M_t} = \frac{1-k_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln M_t},$$

$$(49) \quad \frac{d \ln \tau_{Mt}}{d \ln M_t} = \frac{1-\tau_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln M_t}.$$

Proof: See Appendix 1.

The mechanism for propositions 4 and 5 goes as follows. *Ceteris paribus*, if $\varepsilon < 1$, technological progress in one sector induces a more than proportionate relative price drop (compared to the relative marginal product of labor rise) in the same sector. Hence labor shifts out this sector to preserve the wage parity condition (35). Capital and land use shift in the same direction due to their complementarity during sectoral production. These two propositions correspond to “Baumol’s cost disease” being highlighted in Ngai and Pissarides (2007)’s paper: production inputs move in the direction of the relatively technological stagnating sector.

Proposition 6 (Capital deepening effect): In a competitive equilibrium,

(50)

$$\frac{d \ln l_{Mt}}{d \ln K_t} = - \frac{(1-\varepsilon)(\beta_M - \beta_A)(1-l_{Mt})}{\varepsilon + (1-\varepsilon)[\alpha_M(1-l_{Mt}) + \alpha_A l_{Mt} + \beta_M(1-k_{Mt}) + \beta_A k_{Mt} + \gamma_M(1-\tau_{Mt}) + \gamma_A \tau_{Mt}]} < 0 \quad \text{if } \varepsilon < 1 \text{ and } \beta_M > \beta_A$$

$$> 0 \quad \text{if } \varepsilon < 1 \text{ and } \beta_M < \beta_A$$

(51) $\frac{d \ln k_{Mt}}{d \ln K_t} = \frac{1-k_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln K_t},$

(52) $\frac{d \ln \tau_{Mt}}{d \ln K_t} = \frac{1-\tau_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln K_t}.$

Proof: See Appendix 1.

The mechanism for proposition 6 is similar to those in propositions 3-5. *Ceteris paribus*, if $\varepsilon < 1$, capital deepening induces a more than proportionate relative price drop (compared to the relative marginal product of capital rise) in the sector with higher capital intensity. Hence capital shifts out this sector to retain the capital rental parity condition (36). Labor and land use also move in the same direction. This is the channel highlighted by Acemoglu and Guerrieri (2008): capital deepening leads to factor reallocation towards the sector with lower capital intensity.

To summarize, given $\varepsilon < 1$, the above four mechanisms all work through the relative price effect that dominates over the relative marginal product effect. Population growth effect pushes production factors towards the sector with stronger diminishing returns to labor.²⁶ Technology growth effects push factors towards the sector experiencing slower technological progress. Capital deepening effect pushes factors towards the sector with lower capital intensity.²⁷

5.3 Intertemporal level: Consumption-saving across time

In the intertemporal level, at each time point t , the social planner solves the consumption-saving problem to maximize the objective function:

²⁶ Note that population growth effect depends on the difference between degrees of diminishing returns to labor in the two sectors ($(\alpha_M - \alpha_A)$ in (41)), but not the difference between land intensities between the two sectors ($\gamma_M - \gamma_A$). So a statement like “population growth effect pushes production factors towards the sector with higher land intensity” is not precise, and sometimes incorrect.

²⁷ We might also consider how an exogenous increase in land supply could contribute to a “land expansion effect” on structural transformation. Such effect might have contributed to agricultural-to-manufacturing transformation in the United States during AD1790-AD1870. See Appendix 2 for details.

$$(53) \quad \max_{\{\tilde{c}_t, K_t\}_{t=0}^{\infty}} \int_0^{\infty} e^{-(\rho-n)t} \left(\frac{\tilde{c}_t^{1-\theta}-1}{1-\theta} \right) dt, \text{ subject to}$$

$$(54) \quad \dot{K}_t = \Phi(K_t, t) - \delta K_t - e^{nt} L_0 \tilde{c}_t,$$

where $\Phi(K_t, t)$ is the maximized value of current output at time t (equation (34)), which is a function of the capital stock at time t :

$$\Phi(K_t, t) \equiv \max_{L_{At}, L_{Mt}, K_{At}, K_{Mt}, T_{At}, T_{Mt}} Y_t, \text{ given } K_t > 0.$$

Note that $\Phi(K_t, t)$ contains trending variables such as L_t and M_t (or A_t), and sectoral shares l_{Mt} , k_{Mt} and τ_{Mt} which evolve over time.²⁸

Maximizing (53) subject to (54) is a standard optimal control problem. It yields the consumption Euler equation:

$$(55) \quad \frac{\dot{\tilde{c}}_t}{\tilde{c}_t} = \frac{1}{\theta} [\Phi_K - \delta - \rho],$$

where Φ_K is the marginal product of capital of the maximized production function, which equals the capital rental R_t in the economy. Equations (55) and (54) characterize how per capita consumption composite and aggregate capital stock evolve over time.

To characterize the equilibrium dynamics of the system, we need to impose certain assumptions, appropriately normalize per capita consumption composite and aggregate capital stock, and include sectoral share evolution equations.²⁹ For the first purpose, we assume that:

$$(A1) \quad \varepsilon < 1,$$

$$(A2) \quad \beta_M > \beta_A,$$

$$(A3) \quad g_A > \left(\frac{1-\beta_A}{1-\beta_M} \right) g_M + \left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M - \beta_A)}{1-\beta_M} \right] n.$$

Assumption (A1) states that agricultural and manufacturing goods are consumption complements. Assumption (A2) states that the manufacturing sector is the capital-intensive sector in the economy.

We denote $\left(\frac{1-\beta_A}{1-\beta_M} \right) g_M$ as the augmented manufacturing technology growth rate, and $\left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M - \beta_A)}{1-\beta_M} \right] n$ as the augmented population growth rate. Assumption (A3) states that the

agricultural technology growth rate is greater than the sum of augmented manufacturing technology growth rate and augmented population growth rate (we will explain this assumption in more detail in section 5.4). These three assumptions assure that the manufacturing sector is the asymptotically dominant sector.³⁰

For the second purpose, we normalize per capita consumption composite and aggregate capital stock by population and productivity of the asymptotically dominant sector:

²⁸ See equation (A.8) in Appendix 1 for the reduced-form expression of $\Phi(K_t, t)$.

²⁹ Mathematically, we want to remove the trending terms in (54)-(55) and include a sufficient number of equations to capture the evolution of per capita consumption composite, aggregate capital stock and sectoral shares in an autonomous system of differential equations.

³⁰ We adopt Acemoglu and Guerrieri (2008, 479)'s notation that "[t]he asymptotically dominant sector is the sector that determines the long-run growth rate of the economy."

$$(56) \quad c_t \equiv \frac{\tilde{c}_t l_t^{\frac{1-\alpha_M-\beta_M}{1-\beta_M}}}{M_t^{\frac{1}{1-\beta_M}}} ,$$

$$(57) \quad \chi_t \equiv \frac{K_t^{\frac{1-\beta_M}{\alpha_M}}}{L_t M_t^{\frac{1}{\alpha_M}}} .$$

With these two normalized variables, given the initial conditions χ_0 and k_{M0} , we can characterize the equilibrium dynamics of the economy by an autonomous system of three differential equations in c_t , χ_t and k_{Mt} , as stated in proposition 7.

Proposition 7 (Equilibrium dynamics): Suppose (A1)-(A3) hold. The equilibrium dynamics of the economy is characterized by the following three differential equations:

$$(58) \quad \frac{\dot{c}_t}{c_t} = \frac{1}{\theta} \left(\omega_M \beta_M \eta_t^\varepsilon \chi_t^{-\alpha_M} T^{\gamma_M} l_{Mt}^{\alpha_M} k_{Mt}^{\beta_M-1} \tau_{Mt}^{\gamma_M} - \delta - \rho \right) - \frac{g_M}{1-\beta_M} + \left(\frac{1-\alpha_M-\beta_M}{1-\beta_M} \right) n ,$$

$$(59) \quad \frac{\dot{\chi}_t}{\chi_t} = \frac{1-\beta_M}{\alpha_M} \left(\eta_t \chi_t^{-\alpha_M} T^{\gamma_M} l_{Mt}^{\alpha_M} k_{Mt}^{\beta_M} \tau_{Mt}^{\gamma_M} - \chi_t^{-\frac{\alpha_M}{1-\beta_M}} \cdot c_t - \delta \right) - n - \frac{g_M}{\alpha_M} ,$$

$$(60) \quad \frac{\dot{k}_{Mt}}{k_{Mt}} = \frac{(1-k_{Mt}) \left\{ (g_M - g_A) + (\alpha_M - \alpha_A) n + (\beta_M - \beta_A) \left[\frac{\alpha_M}{1-\beta_M} \chi_t + \frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} \right] \right\}}{\left(\frac{\varepsilon}{\varepsilon-1} \right) - [(\beta_M - \beta_A)(1-k_{Mt}) + (\alpha_M - \alpha_A)(1-l_{Mt}) + (\gamma_M - \gamma_A)(1-\tau_{Mt}) + \alpha_A + \beta_A + \gamma_A]} ,$$

where

$$(61) \quad \eta_t \equiv \omega_M^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{\beta_M}{\beta_A} \right) \left(\frac{1-k_{Mt}}{k_{Mt}} \right) \right]^{\frac{\varepsilon}{\varepsilon-1}} ,$$

given $\chi_0, k_{M0} > 0$, and the transversality condition is satisfied:

$$(62) \quad \lim_{t \rightarrow \infty} \exp \left(\left\{ -\rho + \left[\frac{\alpha_M + (1-\alpha_M-\beta_M)\theta}{1-\beta_M} \right] n + \left(\frac{1-\theta}{1-\beta_M} \right) g_M \right\} t \right) c_t^{-\theta} \chi_t^{\frac{\alpha_M}{1-\beta_M}} = 0 .$$

Proof: See Appendix 1.

The dynamic system (58)-(60) in proposition 7 is a three-dimensional generalization of the per capita consumption-effective capital-labor ratio dynamic system in Ramsey (1928)-Cass (1965)-Koopmans (1965) model, where we add in features of sectoral production and land as a fixed production factor.³¹ Note that l_{Mt} and τ_{Mt} in (58)-(60) are functions of k_{Mt} at each time t (see intratemporal equilibrium conditions (39)-(40)). They evolve according to:

$$(63) \quad \frac{\dot{l}_{Mt}}{l_{Mt}} = \left(\frac{1-l_{Mt}}{1-k_{Mt}} \right) \frac{\dot{k}_{Mt}}{k_{Mt}} ,$$

$$(64) \quad \frac{\dot{\tau}_{Mt}}{\tau_{Mt}} = \left(\frac{1-\tau_{Mt}}{1-k_{Mt}} \right) \frac{\dot{k}_{Mt}}{k_{Mt}} .$$

We give the interpretations of the above equations: (58) and (59) are the consumption Euler equation and capital accumulation equation transformed to sort out the trending population and productivity terms; (60), (63) and (64) come from taking log and differentiating the intratemporal equilibrium conditions (38)-(40), and they show how the sectoral shares evolve over time.³² We

³¹ Setting $\gamma_M = 0$, $k_{Mt} = l_{Mt} = \tau_{Mt} = 1$ reduces (56)-(62) to Ramsey (1928)-Cass (1965)-Koopmans (1965) model's two-dimensional dynamic equation system.

³² For relative sectoral output evolution, equation (A.1) in Appendix 1 still holds in the unified model. That is, given $\varepsilon < 1$, $\frac{\dot{Y}_{Mt}}{Y_{Mt}} - \frac{\dot{Y}_{At}}{Y_{At}}$ and $\frac{\dot{l}_{Mt}}{l_{Mt}}$ follow different signs.

impose the following parameter restriction to guarantee the transversality condition (62):

$$(A4) \quad \rho - \left[\frac{\alpha_M + (1 - \alpha_M - \beta_M)\theta}{1 - \beta_M} \right] n > \left(\frac{1 - \theta}{1 - \beta_M} \right) g_M .$$

5.4 Constant growth path (CGP)

We focus on one particular equilibrium path characterized by proposition 7: the constant growth path (CGP), which is defined as a path featured with constant normalized per capita consumption composite growth rate. Later we will state that the equilibrium dynamics of the economy converges to the CGP (section 5.5). Proposition 8 shows the closed-form solution of sectoral share evolution equations in CGP.

Proposition 8 (Structural transformation in CGP): Suppose (A1)-(A4) hold. In a constant growth path, sectoral shares evolve according to:

$$(65) \quad \frac{k_{Mt}}{k_M} = G(k_{Mt}) \left\{ g_A - \left(\frac{1 - \beta_A}{1 - \beta_M} \right) g_M - \left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M - \beta_A)}{1 - \beta_M} \right] n \right\} ,$$

where $G(k_{Mt}) > 0$ is a function of k_{Mt} and is unrelated to g_A , g_M and n ;

and (63)-(64), given $k_{M0} > 0$.

As $t \rightarrow \infty$, $k_{Mt} \rightarrow k_M^* = 1$, $l_{Mt} \rightarrow l_M^* = 1$ and $\tau_{Mt} \rightarrow \tau_M^* = 1$.

Proof: See Appendix 1.

Proposition 8 highlights the result of interplay among population growth effect, technology growth effects and capital deepening effect in fostering structural transformation in CGP. Equation (65) explains why assumption (A3) guarantees that the manufacturing sector is the asymptotically dominant sector: we need a strong enough agricultural technology growth effect which overrides manufacturing technology growth effect, population growth effect and capital deepening effect to ensure factor reallocations towards the manufacturing sector. The technology growth effects from the two sectors are represented by the g_A and g_M terms. The population growth effect is represented by the $[\alpha_M - \alpha_A]n$ term. Capital accumulation is endogenous in the model and the capital deepening effect is captured by the “wedge” coefficients $\frac{1 - \beta_A}{1 - \beta_M}$ and $\frac{\alpha_M(\beta_M - \beta_A)}{1 - \beta_M}$, which respectively augment the manufacturing technology growth effect and population growth effect terms relative to the agricultural technology growth effect term. We reinforce our result in the following corollary.

Corollary 8 (Escape from land): In the unified model, suppose there are two sectors producing consumption complements in the economy: one is land-intensive and the other is capital-intensive. Production factors shift from the land-intensive sector to the capital-intensive sector if the technology growth rate in the land-intensive sector is greater than the sum of augmented technology growth rate in the capital-intensive sector and augmented population growth rate.³³

³³ Due to model symmetry, suppose instead the agricultural sector is capital-intensive ($\beta_M < \beta_A$) and the manufacturing sector is land-intensive ($\gamma_M > \gamma_A$). Given that the two sectors produce consumption complements, the condition to ensure “escape from land” is $g_M > \left(\frac{1 - \beta_M}{1 - \beta_A} \right) g_A + \left[\alpha_A - \alpha_M + \frac{\alpha_A(\beta_A - \beta_M)}{1 - \beta_A} \right] n$.

Corollary 8 highlights structural transformation in an economy that features population growth, technological progress and capital accumulation. Given agriculture is the land-intensive sector, the key to move production factors out of agriculture is agricultural productivity growth itself (see the United States example in section 6.2).³⁴

Next, we investigate a “razor’s edge” condition. From (65), unless the following “razor’s edge” condition holds:

$$(66) \quad \alpha_M - \alpha_A + \frac{\alpha_M(\beta_M - \beta_A)}{1 - \beta_M} = 0, \quad ,$$

otherwise a change in population growth rate will affect the direction and pace of structural transformation in CGP. The “razor’s edge” condition (66) can be reduced to either $\gamma_A = \gamma_M = 0$ or $\frac{\alpha_A}{\alpha_M} = \frac{\gamma_A}{\gamma_M}$. The former means land intensities equal zero in the two sectors. The latter means the ratio of labor intensity equals the ratio of land intensity in the two sectors.

Acemoglu and Guerrieri (2008)’s model is a special case of ours, where the “razor’s edge” condition (66) applies. In their paper, they did not include land as an input in sectoral production. This is equivalent to setting $\gamma_A = \gamma_M = 0$, $\alpha_A = 1 - \beta_A$ and $\alpha_M = 1 - \beta_M$ in our model. Equation (65) is reduced to $\frac{\dot{k}_{Mt}}{k_{Mt}} = G(k_{Mt}) \left\{ g_A - \left(\frac{\alpha_A}{\alpha_M} \right) g_M \right\}$. It happens that the population growth effect is cancelled out by some part of the capital deepening effect, and population growth rate does not show up in the sectoral share evolution equation. Also, as a special case of our (A3), they assume $g_A - \left(\frac{\alpha_A}{\alpha_M} \right) g_M > 0$ to make sure that the manufacturing sector is the asymptotically dominant sector.³⁵

Our model can also be collapsed to the two-sector version of Ngai and Pissarides (2007)’s one, which again fulfils the “razor’s edge” condition. In their paper, land is not an input to sectoral production. They also assumed same capital intensity for all sectoral production functions. This makes $\gamma_A = \gamma_M = 0$, $\alpha_A = 1 - \beta_A$, $\alpha_M = 1 - \beta_M$, $\alpha_A = \alpha_M$ and $\beta_A = \beta_M$ in our model. Equation (65) is reduced to $\frac{\dot{k}_{Mt}}{k_{Mt}} = G(k_{Mt}) \{g_A - g_M\}$. There was neither population growth effect nor capital deepening effect in the reduced model. By assuming $g_A - g_M > 0$, we get their result that the sector with the slowest technology growth will continuously draw in employment in the aggregate balanced growth path.³⁶

³⁴ The policy implication of corollary 8 is that, to foster industrialization, it is important to assure agricultural productivity growth. During the Great Leap Forward years in China (AD1958-AD1961), the fall in agricultural productivity (Lin 1990) would be a reason behind the failure of the government-led industrialization, which ended up with a severe famine (Zhu 2012). On the other hand, the low or even negative population growth rates in Japan and EU countries (Maddison 2008) would have been fostering industrialization in the recent decades, by making (A3) more likely to hold.

³⁵ See Proposition 3 (sectoral share evolution equation) and Assumption 2(i) in Acemoglu and Guerrieri (2008)’s paper.

³⁶ See Proposition 2 in Ngai and Pissarides (2007)’s paper. Note that Ngai and Pissarides (2007) have examined the inclusion of a fixed production factor in at least one production sector in their appendix.

From (65), we can also study the effects of changes in technology growth rates and population growth rate on the pace of agricultural-to-manufacturing transformation, given that (A1)-(A4) hold. Straightforward differentiation yields:

$$(67) \quad \frac{d\left(\frac{k_{Mt}}{k_{Mt}}\right)}{dg_A} = G(k_{Mt}) > 0 ,$$

$$(68) \quad \frac{d\left(\frac{k_{Mt}}{k_{Mt}}\right)}{dg_M} = -G(k_{Mt}) \left(\frac{1-\beta_A}{1-\beta_M}\right) < 0 ,$$

$$(69) \quad \frac{d\left(\frac{k_{Mt}}{k_{Mt}}\right)}{dn} = -G(k_{Mt}) \left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M - \beta_A)}{1-\beta_M} \right] \geq 0 .$$

Speeding up agricultural technological progress accelerates sectoral shift, while boosting manufacturing technology growth rate decelerates it. Increasing population growth rate has a theoretically ambiguous effect on the pace of sectoral shift, and we resolve the sign by relying on the estimates of sectoral production function parameters. We consider the agricultural and manufacturing production functions calibrated by Gollin et al. (2007):³⁷

$$(70) \quad Y_{At} = A_t L_{At}^{0.6} K_{At}^{0.1} T_{At}^{0.3} , \text{ that is, } \alpha_A = 0.6, \beta_A = 0.1, \gamma_A = 0.3 .$$

$$(71) \quad Y_{Mt} = M_t L_{Mt}^{0.5} K_{Mt}^{0.5} , \text{ that is, } \alpha_M = 0.5, \beta_M = 0.5, \gamma_M = 0 .$$

Plug the coefficients from (70) and (71) into (69) to get $\frac{\partial\left(\frac{k_{Mt}}{k_{Mt}}\right)}{\partial n} = -0.3 \cdot G(k_{Mt}) < 0$. An increase in population growth rate will slow down structural transformation.³⁸

5.5 Asymptotic growth path

Lastly, we study the properties of the economy in its asymptotic growth path. The economy converges to a unique, saddle-path stable CGP with non-balanced sectoral growth, which is summarized in proposition 9.

Proposition 9 (Asymptotic growth path): Suppose (A1)-(A4) hold, denote $a^* \equiv \lim_{t \rightarrow \infty} a_t$, $g_a^* \equiv \lim_{t \rightarrow \infty} \left(\frac{\dot{a}_t}{a_t}\right)$, $y_t \equiv \frac{Y_t}{L_t}$ as per capita final output or per capita income in the economy, then there exists a unique, saddle-path stable asymptotic growth path such that:

$$k_M^* = l_M^* = \tau_M^* = 1, \eta^* = \omega_M^{\frac{\varepsilon}{\varepsilon-1}} .$$

$$\chi^* = \left[\frac{\theta g_M - (1-\alpha_M-\beta_M)\theta n + \delta + \rho}{\beta_M \omega_M^{\frac{\varepsilon}{\varepsilon-1} T^{\gamma_M}}} \right]^{\frac{1}{\alpha_M}} , \quad c^* = \left[\eta^* (\chi^*)^{-\alpha_M T^{\gamma_M}} - \delta - \frac{\alpha_M}{1-\beta_M} \left(n + \frac{g_M}{\alpha_M} \right) \right] (\chi^*)^{\frac{\alpha_M}{1-\beta_M}} .$$

For the aggregate variables,

³⁷ Gollin et al. (2007, 1237) stated the non-agricultural production function in the form of $Y_{Mt} = M_t L_{Mt}^{0.5} K_{Mt}^{0.5} + \sigma L_{Mt}$, where σ is a small positive number to guarantee that an economy without initial capital will accumulate capital. We do not need this technical assumption as we have assumed initial capital to be greater than zero (proposition 7); we directly set $\sigma = 0$ to obtain (71).

³⁸ In other words, our model implies that an increase in population growth rate would slow down economic development in terms of counteracting agricultural technology growth effect, retaining production factors in agriculture. This is in analogy to unified growth theories' mechanism in which an increase in population growth rate would neutralize the effect of technological progress, hence retain per capita income in a Malthusian Trap (Galor and Weil 2000; Galor and Moav 2002).

$$g_Y^* = \frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} ,$$

$$g_y^* = \frac{g_M}{1-\beta_M} - \left(\frac{1-\alpha_M-\beta_M}{1-\beta_M} \right) n ,$$

$$g_c^* = \frac{g_M}{1-\beta_M} - \left(\frac{1-\alpha_M-\beta_M}{1-\beta_M} \right) n ,$$

$$g_L^* = n ,$$

$$g_K^* = \frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} ,$$

$$g_T^* = 0 .$$

For the agricultural sector,

$$g_{Y_A}^* = g_A + \alpha_A n + \beta_A \left(\frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} \right) ,$$

$$g_{L_A}^* = \left(1 - \frac{1}{\varepsilon} \right) (g_{Y_A}^* - g_{Y_M}^*) + n ,$$

$$g_{K_A}^* = \left(1 - \frac{1}{\varepsilon} \right) (g_{Y_A}^* - g_{Y_M}^*) + \frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} ,$$

$$g_{T_A}^* = \left(1 - \frac{1}{\varepsilon} \right) (g_{Y_A}^* - g_{Y_M}^*) .$$

For the manufacturing sector,

$$g_{Y_M}^* = \frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} < g_{Y_A}^* ,$$

$$g_{L_M}^* = n > g_{L_A}^* ,$$

$$g_{K_M}^* = \frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} > g_{K_A}^* ,$$

$$g_{T_M}^* = 0 > g_{T_A}^* .$$

Proof: See Appendix 1.

There is non-balanced growth in the sense that the manufacturing sector will tend to draw away all production resources (capital, labor and land) in the economy and become the asymptotically dominant sector. On the other hand, the agricultural output will grow at a faster rate than the manufacturing output. Similar to the basic model (proposition 2), given $\alpha_M + \beta_M \neq 1$, population growth puts a drag $-\left(\frac{1-\alpha_M-\beta_M}{1-\beta_M} \right) n$ on per capita income growth rate in the asymptotic growth path. This population growth drag again originates from the fixed land supply and the associated diminishing returns to labor during production process.³⁹ Although this drag is likely to be quantitatively small (due to the low land intensity in manufacturing production in reality), our result still yields new theoretical insight not evident in the literature (section 2). To ensure a sustainable per capita income growth, technological progress in the asymptotically dominant

³⁹ In Ramsey (1928)-Cass (1965)-Koopmans (1965) model, $\gamma_M = 0$ and so $-\left(\frac{1-\alpha_M-\beta_M}{1-\beta_M} \right) n = 0$. There would hence be no population growth drag for g_y^* and g_c^* terms.

sector needs to outpace the population growth drag.⁴⁰

6 TWO SIMPLE SIMULATIONS

6.1 Structural Transformation in pre-industrial England (AD1521-AD1745)

Consider a slight modification to the basic (two-sector, two-factor) model in section 4. We allow for agricultural technological progress to take place at a rate $g_A \geq 0$. Equation (17) is modified to:⁴¹

$$(72) \quad \frac{l_{Mt}}{l_{Mt}} = \frac{[-g_A + (\alpha_M - \alpha_A)n](1 - l_{Mt})}{\frac{\varepsilon}{\varepsilon - 1} - [(\alpha_M - \alpha_A)(1 - l_{Mt}) + (\gamma_M - \gamma_A)(1 - \tau_{Mt}) + \alpha_A + \gamma_A]},$$

and (18) remains valid. We apply this model to pre-industrial England (AD1521-AD1745). We believe the modified model is a good representation of England's economic environment for this time frame. England was in its late-Malthusian era and began the Agricultural Revolution in the seventeenth century. Gradual population growth and accelerating agricultural technological progress were featuring this period, while manufacturing technological progress and capital accumulation had not speeded up yet.⁴²

Following Judd (1998, ch.10), we employ the Euler method to discretize (72) and (18) into difference equations in l_{Mt} and τ_{Mt} . Together with $l_{At} = (1 - l_{Mt})$ and $\tau_{At} = (1 - \tau_{Mt})$, we have a system of difference equations in four unknowns l_{At} , l_{Mt} , τ_{At} and τ_{Mt} . Each model period corresponds to one year.

Table 2 shows the benchmark parameters and initial values we use in this subsection. We adjust $\alpha_A = 0.4$, $\gamma_A = 0.6$, $\alpha_M = 1$ and $\gamma_M = 0.01$ to match the agricultural and manufacturing production functions calibrated by Vollrath (2011) and Yang and Zhu (2013).⁴³ We set the initial agricultural labor share as 0.581 (Broadberry et al. 2013). There were no estimates for England's sectoral land shares in those time periods. We let the initial agricultural land share to be 0.95.⁴⁴ The agricultural technology growth rates come from Clark (2002)'s estimated annual agricultural productivity growth rates (Table 1). We assume there was no growth in manufacturing technology throughout the simulation time frame. For AD1551-AD1745, we calculate the

⁴⁰ Mathematically, we require $g_M > (1 - \alpha_M - \beta_M)n$ to ensure a sustainable per capita income growth in the asymptotic growth path.

⁴¹ For deriving (72), see the last part in the proof of proposition 1 in Appendix 1.

⁴² The Agricultural Revolution took place in Britain during AD1600-AD1750 (Allen 2004). The British Industrial Revolution occurred in the late-eighteenth century (Ashton 1948). In AD1760-AD1800, total factor productivity in manufacturing grew slowly, at around 0.2% per annum (Crafts 1985, 84). We assume there was no manufacturing technological progress before AD1760. For capital accumulation, the share of gross national income devoted to gross domestic investment was low before the British Industrial Revolution, at 4-6% in AD1700-AD1760 (Crafts 1985, 73).

⁴³ We set γ_M to be a value slightly greater than zero so that land is an essential input to manufacturing production. Otherwise there will never be land allocated to the manufacturing sector.

⁴⁴ The World Bank (2016) provided the agricultural land share data since the late-twentieth century. The British agricultural land share equals 0.82 in AD1961. We hypothesize the agricultural land share in England in AD1521 to be greater than this value and we let it be 0.95.

population growth rates using Mitchell (1988)’s population estimates. For AD1521-AD1550, Pamuk (2007)’s AD1400 population estimate and Mitchell’s AD1550 population estimate implied there was a negligible population growth within this time frame (Figure 2). We set $\varepsilon = 0.5$ (Buera and Kaboski 2009).

[INSERT TABLE 2 HERE]

Figure 8 (solid lines) depicts the benchmark simulation result. Now we explain the inverted U-shaped agricultural labor share evolution over AD1521-AD1745 in the left panel. During AD1521-AD1550 there were negligible population growth and agricultural technological progress. This resulted in weak population growth effect and agricultural technology growth effect, retaining the agricultural labor share at a roughly constant (or slowly rising) level. During AD1551-AD1605 population growth accelerated, and through population growth effect labor “embraced” the agricultural sector (Corollary 1). Since AD1605 agricultural technology growth picked up (the Agricultural Revolution), neutralizing the population growth effect and decelerating the rise in agricultural labor share. During AD1661-AD1745 population growth slowed down and the agricultural technology growth effect dominated, causing labor to shift out of agriculture.⁴⁵ On the other hand, due to input complementarity, the agricultural land share also followed an inverted U-shaped trend throughout the simulation time frame in the right panel. However, our simulation predicts too small a drop in agricultural labor share at least since the AD1650s. Population growth and agricultural technological progress are not sufficient to quantitatively reconcile structural transformation in England since the seventeenth century.

[INSERT FIGURE 8 HERE]

Still, the population growth effect is important for us to reconcile the rise in agricultural labor share during the sixteenth century. To illustrate this, we perform a counterfactual experiment. The dashed lines in Figure 8 depict the simulated paths by adopting all benchmark parameters and initial values in Table 2, except resetting the population growth rate to zero ($n = 0$) for the entire simulation time frame; that is, the population growth effect is completely shut down. The counterfactual exercise fails to reconcile quantitatively the rise in agricultural labor share by the AD1570s. We conclude that population growth effect is a key determinant of sectoral labor share evolution in England during the early Modern Period, but it (together with agricultural technology growth effect) is not adequate in accounting for structural transformation since the seventeenth century.⁴⁶

6.2 *Structural Transformation in the modern United States (AD1980-AD2100)*

In this subsection we apply the unified (two-sector, three-factor) model in section 5 to the modern United States (AD1980-AD2100). Technological progress and capital accumulation have become significant features in modern economic growth. We first examine whether the United States was characterized by CGP, so that we can apply the model equations (63)-(65).

⁴⁵ The dots in the left panel of Figure 8 reproduce Clark (2010, 2013)’s agricultural labor share estimates from Figure 1.

⁴⁶ See section 7 for a potential explanation to account for the rapid drop in English agricultural labor share in AD1661-AD1745.

Figure 9 depicts the annual growth rate of normalized real per capita consumption expenditure series (solid line) and its ten-year average series (dashed line) in the United States during AD1948-AD2002.⁴⁷ Since the AD1980s, the ten-year average series has stayed at a roughly constant level of 1%. Therefore we accept that the United States was growing along a CGP since the AD1980s, and choose AD1980 as the starting year for simulation. Note another reason for choosing AD1980 is that, only after this year do we have agricultural labor share estimates from the World Bank (2016).

[INSERT FIGURE 9 HERE]

Again, we employ the Euler method to discretize (63)-(65) into difference equations in k_{Mt} , l_{Mt} and τ_{Mt} . Together with $l_{At} = (1 - l_{Mt})$, $k_{At} = (1 - k_{Mt})$ and $\tau_{At} = (1 - \tau_{Mt})$, we have a system of difference equations in six unknowns k_{At} , k_{Mt} , l_{At} , l_{Mt} , τ_{At} and τ_{Mt} . Each model period represents a year.

Table 3 shows the baseline parameters and initial values we employ in this subsection. We follow Gollin et al. (2007) and let the sectoral production functions to take the forms of (70) and (71). We set initial agricultural labor and land shares as the World Bank (2016)'s AD1980 estimates, and the initial agricultural capital share as BEA (2016)'s estimate of proportion of private fixed assets held by agriculture, forestry, fishing and hunting sectors in AD1980 (Figure 3). We calculate the population growth rate to match the annualized growth rate of full-time and part-time employees throughout AD1980-AD2002 (Figure 6), provided by BEA (2016).⁴⁸ Next we set agricultural technology growth rate as the annualized growth rate of farm total factor productivity over AD1980-AD2002 (Figure 4), provided by U.S. Department of Agriculture (2016), and the manufacturing technology growth rate as the annualized multifactor productivity growth rate for private nonfarm business sector during AD1980-AD2002 (Figure 5), provided by BLS (2016). Lastly we fix $\varepsilon = 0.5$ (Buera and Kaboski 2009).

[INSERT TABLE 3 HERE]

Figure 10 (solid lines) depicts the baseline simulation result. There are four points to note. First, the agricultural capital share (left panel), labor share (middle panel) and land share (right panel) were falling throughout the simulation time frame, which qualitatively matches the trends showed in Figure 3. The underlying reason is because (A1)-(A3) were satisfied at the parameter values given in Table 3. "Escape from land" took place (Corollary 8): factor inputs continuously shifted from the agricultural to the manufacturing sector, and the economy endogenously transformed to a production mode that is less land-intensive.⁴⁹

⁴⁷ The normalized real per capita consumption expenditure series is constructed from the real personal consumption expenditures per capita data provided by Federal Reserve Bank of St. Louis (2016), normalized by population and manufacturing technology levels according to (56).

⁴⁸ We calibrate the parameters using data and estimates within AD1980-AD2002. We choose AD2002 as the terminating point because it was the last year BLS provided the multifactor productivity estimate for the private nonfarm business sector using the SIC (Standard Industrial Classification) system (Figure 5). Actually starting in AD2015, BLS provided a new "historical multifactor productivity measures (SIC 1948-87 linked to NAICS 1987-2013)" series, which blended multifactor productivity estimates using two classification systems (SIC and NAICS). However, even if we are willing to overpass the changing system issue, the longer time series showed that normalized real per capita consumption expenditure has not followed a CGP since the mid-AD2000s. Therefore we keep our calibration focused on the AD1980-AD2002 time frame.

⁴⁹ Schultz (1953, 127-128) proposed two propositions to represent the historical declining

INSERT FIGURE 10 HERE

Second, our simulation quantitatively matches the fall in agricultural land share in the United States during AD1980-AD2002. We consider this a major success of the unified model. Traditional growth and structural transformation models do not include land as a production factor and thereby cannot reconcile land use reallocation throughout development process.⁵⁰ Our work shows that the relative price effects (propositions 3-6) are sufficient to explain land use movement in the United States in the recent decades.

However, the same cannot be said of capital and labor movements. The simulated agricultural capital and labor shares stay well above their empirical counterparts in AD2002 (depicted by dots in Figure 10). It is not sufficient to focus only on the relative price effects brought about by population growth, technological progress and capital deepening to quantitatively reconcile capital and labor movements. We might need to take the income effect, other supply-side channels or institutions (section 2) into account to quantitatively explain the evolution of these sectoral shares.

Fourth, we perform a counterfactual experiment to illustrate that neglect of population growth significantly affects prediction on the pace of structural transformation in the modern United States. The dashed lines in Figure 10 depict the simulated paths by adopting all baseline parameters and initial values in Table 3, but adjusting $n = 0$ for all time periods. We observe significant divergences in the predicted rates of sectoral shifts between the baseline and counterfactual cases. Ignoring population growth significantly speeds up agricultural-to-manufacturing transformation.

7 DISCUSSION

In this section, we highlight four points of discussion. The first is about the elasticity of substitution term ε . Recall from the concluding paragraph in section 5.2 that, given that two sectoral goods are consumption complements ($\varepsilon < 1$), population growth, technology growth and capital deepening will push production factors away from the sector with weaker diminishing returns to labor, faster technological progress and higher capital intensity. The crux importance of $\varepsilon < 1$ is making sure that the relative price effect (equation (20)) dominates over the relative marginal product effect (equation (21)). This assumption has been explicitly stated in Ngai and

economic importance of land that has characterized Western communities:

“1. A declining proportion of the aggregate inputs of the community is required to produce (or to acquire) farm products.

2. Of the inputs employed to produce farm products, the proportion represented by land is not an increasing one ...

[W]henver both of these propositions are valid, land will necessarily decline in importance in the economy.”

Our model provides a theoretical foundation under which Schultz’s first proposition is valid (when (A1)-(A4) in our model hold).

⁵⁰ Hansen and Prescott (2002), Leukhina and Turnovsky (2016) did not include land as a fixed production factor in Solow/manufacturing sector in their models. In equilibrium there will never be land allocated for manufacturing use. Hence their models cannot reconcile land use reallocation.

Pissarides (2007, 2008), Acemoglu and Guerrieri (2008) and Buera and Kaboski (2009)’s papers, allowing sectors with slower productivity growth and lower capital intensity to draw in production inputs throughout economic development. On the other hand, if ε is sufficiently large, the relative marginal product effect would outweigh the relative price effect, reversing the directions of sectoral shifts in propositions 1-6.⁵¹ Hansen and Prescott (2002), Doepke (2004) and Lagerlöf (2010) implicitly assumed perfect consumption substitutability between two sectoral goods ($\varepsilon \rightarrow \infty$). Given the parameter assumptions in their papers, sectors with faster technological progress will attract production factors throughout development process. Despite the parameter ε being so crucial in explaining structural transformation through supply-side channels, to our knowledge, there is still no well-accepted estimate established for the elasticity of substitution between agricultural and manufacturing goods.

Second, we make a note of English sectoral shift in our simulation in section 6.1. Our simulation significantly under-predicts the drop in agricultural labor share during AD1661-AD1745. The simulated fall is so slow because the agricultural technology growth effect was largely neutralized by the population growth effect, leaving a weak net relative price effect pushing labor out of agriculture. Actually the weak relative price effect fits into the historical evidence. Figure 11 depicts the evolution of relative agricultural price in England during AD1500-AD1800.⁵² There was no obvious trend within AD1661-AD1745. Also, within this time frame, England was stuck in the Malthusian Trap when there was little progress in per capita income (Clark 2007). So it seems like neither the relative price effect nor the income effect could explain the proto-industrialization taking place in this time frame. We resort to attributing such a phenomenon to an exogenous preference shift from agricultural to manufacturing goods during the age of British Consumer Revolution (Weatherill 1996).⁵³

INSERT FIGURE 11 HERE

Third, corollary 1 and corollary 8 highlight the possibility of a nation/region going through a two-stage development process: when technology is stagnating, population growth induces production factors to “embrace the land”; later when (agricultural) technology picks up, the production factors will “escape from land”. We have discussed how the United States experienced the second stage in section 6.2, and indeed early United States history seems to have gone through the first stage too. Lindert and Williamson (2016, Figure 1) stated that colonial United States was ruralizing during AD1680-AD1775; its urban share of population was in general

⁵¹ Because of the land (and capital) movement response to population growth, it turns out that $\varepsilon > 1$ is not a sufficient condition to reverse the directions of sectoral shifts as stated in propositions 1-6. The required condition would instead be:

$$\frac{\varepsilon}{\varepsilon-1} > [(\alpha_M - \alpha_A)(1 - l_{Mt}) + (\gamma_M - \gamma_A)(1 - \tau_{Mt}) + \alpha_A + \gamma_A] \text{ for propositions 1-2; and}$$

$$\frac{\varepsilon}{\varepsilon-1} > [\alpha_M(1 - l_{Mt}) + \alpha_A l_{Mt} + \beta_M(1 - k_{Mt}) + \beta_A k_{Mt} + \gamma_M(1 - \tau_{Mt}) + \gamma_A \tau_{Mt}] \text{ for propositions 3-6.}$$

⁵² The relative agricultural price is the agriculture price index divided by industry price index, provided by Broadberry et al. (2011).

⁵³ From her study of nearly 3,000 probate inventories in Britain during AD1675-AD1725, Weatherill (1996) found that there had been significant increases in the number of cooking equipment (saucepans), eating equipment (pewter dishes and plates, earthenware, knives and forks, utensils for hot drinks), textiles (window curtains), looking glasses and clocks within the time frame.

declining. At the same time, the United States population was undergoing “a rapidity of increase almost without parallel in history” (Malthus 1826, 517); in particular the population of the Thirteen Colonies increased from 0.28 million in AD1700 to 2.50 million in AD1775 (McEvedy and Jones 1978, 290). According to our theory, the rapidity of population increase would be a factor contributing to the ruralization through “embrace the land” mechanism during AD1680-AD1775.⁵⁴

Fourth, we take a brief look on cross-country evidence for the population growth effect on structural transformation in the recent decades. We obtain the AD1980 and AD2010 data on agricultural labor and land shares, as well as the population growth rates in 251 countries during AD1980-AD2010 from the World Bank (2016).⁵⁵ Then we compute the annualized growth rates of agricultural labor and land shares during AD1980-AD2010, and regress them against the annualized population growth rates within the same time frame. Figures 12A and 12B depict the regression results. The positive correlations between agricultural labor or land share growth rates and the population growth rate are consistent with the population growth effect on structural transformation (the simple “embrace the land” version): population growth retains the production factors in the farmland. The slower the population growth is, the faster are labor and land use could be released from the agriculture.⁵⁶ Certainly, to establish causality, we require more in-depth country studies or econometric analyses. We leave it as a topic for future research.⁵⁷

INSERT FIGURE 12 HERE

8 CONCLUSION

Population growth induces structural transformation. This paper works out the underlying logic and unearths the crucial assumptions for the claim. We develop dual-economy growth models. Given two sectors that produce consumption complements, population growth pushes production factors towards the sector characterized by stronger diminishing returns to labor through the relative price effect that dominates over the relative marginal product effect.

⁵⁴ Similar argument provides a reason why Chinese agriculture had not released labor by the late-eighteenth century (Voigtländer and Voth 2013). Allen (2009) found that there was little progress in Yangtze agricultural (labor) productivity in AD1620-AD1820. On the other hand, Chinese population rose from 140 million to 435 million in AD1650-AD1850 (McEvedy and Jones 1978, 167). By corollary 1 Chinese labors would “embrace the land” within this time frame.

⁵⁵ The agricultural labor share, agricultural land share and population growth rate refer to the employment in agriculture (% of total employment), agricultural land (% of land area) and population growth (annual %) respectively, provided by the World Bank (2016).

⁵⁶ The positive slopes of the fitted regression lines in Figures 12A and 12B are significant at 10% level. We have repeated the analysis using the AD1990-AD2010 data, when more data is available, and the qualitative results stay the same.

⁵⁷ Recently, empirical analyses have been performed to trace the determinants of structural transformation. For example, Lee and Wolpin (2006) investigated the relative importance of labor supply and demand factors in explaining the growth of the service sector in the United States during AD1950-AD2000. Michaels et al. (2012) employed a micro-founded model with urbanization and structural transformation to track population evolution in the rural and urban areas in the United States during AD1880-AD2000.

Our models provide theoretical foundations to explain structural transformation via the relative price effects originating from population growth (Leukhina and Turnovsky 2016), technological progress (Ngai and Pissarides 2007) and capital deepening channels (Acemoglu and Guerrieri 2008). We clarify the conditions under which production factors “embrace the land” in early development stages and “escape from land” in advanced development stages (Corollary 1 and Corollary 8). We illustrate how pre-industrial England and the modern United States satisfy the conditions and explain the agricultural-manufacturing transformations that have taken place. However, our models still fall short of quantitatively accounting for labor (and capital) movements in specific time periods. This indicates that we still leave out some components which play first-order important roles in determining sectoral shifts. Some potential candidates include the income effect (Kongsamut et al. 2001), the scale effect (Buera and Kaboski 2012a) or other channels we have discussed in the literature review (section 2).

A unified explanation for structural transformation to reconcile the past and modern observations is a challenging and fascinating topic. Future work on combining the relative price effects with the other mechanisms fostering structural transformation to quantitatively reconcile non-balanced economic growth will be a fruitful area of research. Hopefully our analysis also sheds light on broader issues related to economic modeling, income growth and development history.

Appendix 1: Proofs for the propositions

Proposition 1

Proof: Rewrite (15) as $(l_{Mt}^{-1} - 1) = \frac{\omega_A \alpha_A}{\omega_M \alpha_M} \left(\frac{Y_{Mt}}{Y_{At}} \right)^{\frac{1-\varepsilon}{\varepsilon}}$. Take log and differentiate with respect to time to get

$$(A.1) \quad \frac{\dot{l}_{Mt}}{l_{Mt}} = (1 - l_{Mt}) \left(\frac{\varepsilon-1}{\varepsilon} \right) \left(\frac{\dot{Y}_{Mt}}{Y_{Mt}} - \frac{\dot{Y}_{At}}{Y_{At}} \right).$$

Note from (4) and (5), $\frac{\dot{Y}_{Mt}}{Y_{Mt}} - \frac{\dot{Y}_{At}}{Y_{At}} = \alpha_M \left(\frac{\dot{l}_{Mt}}{l_{Mt}} + \frac{\dot{L}_t}{L_t} \right) - \alpha_A \left(-\frac{\dot{l}_{Mt}}{l_{At}} + \frac{\dot{L}_t}{L_t} \right) + \gamma_M \left(\frac{\dot{\tau}_{Mt}}{\tau_{Mt}} \right) - \gamma_A \left(-\frac{\dot{\tau}_{Mt}}{\tau_{At}} \right)$ or

(A.2)

$$\frac{\dot{Y}_{Mt}}{Y_{Mt}} - \frac{\dot{Y}_{At}}{Y_{At}} = (\alpha_M - \alpha_A) \left(\frac{\dot{l}_{Mt}}{l_{Mt}} + \frac{\dot{L}_t}{L_t} \right) + \alpha_A \left(\frac{\dot{l}_{Mt}}{l_{Mt}(1-l_{Mt})} \right) + (\gamma_M - \gamma_A) \left(\frac{\dot{\tau}_{Mt}}{\tau_{Mt}} \right) + \gamma_A \left(\frac{\dot{\tau}_{Mt}}{\tau_{Mt}(1-\tau_{Mt})} \right).$$

Rewrite (16) as $(\tau_{Mt}^{-1} - 1) = \frac{\gamma_A \alpha_M}{\gamma_M \alpha_A} \left(\frac{1-l_{Mt}}{l_{Mt}} \right)$. Take log and differentiate with respect to time to get (18).

Combining (18) with (A.1)-(A.2), we get (17).

From (A.1), $\frac{\dot{Y}_{Mt}}{Y_{Mt}} - \frac{\dot{Y}_{At}}{Y_{At}}$ follows a different sign from $\frac{\dot{l}_{Mt}}{l_{Mt}}$ if $\varepsilon < 1$.

For proof of (72) only: in case there is agricultural productivity growth at a rate g_A , the right hand

side of (A.2) becomes

$$(A.2') \quad \frac{\dot{Y}_{Mt}}{Y_{Mt}} - \frac{\dot{Y}_{At}}{Y_{At}} = -g_A + (\alpha_M - \alpha_A) \left(\frac{l_{Mt}}{l_{Mt}} + \frac{\dot{l}_t}{l_t} \right) + \alpha_A \left(\frac{l_{Mt}}{l_{Mt}(1-l_{Mt})} \right) + (\gamma_M - \gamma_A) \left(\frac{\tau_{Mt}}{\tau_{Mt}} \right) + \gamma_A \left(\frac{\tau_{Mt}}{\tau_{Mt}(1-\tau_{Mt})} \right).$$

Combine (A.2') with (18), then we obtain (72).

Proposition 2

Proof: The first statement follows immediately from (17) and (18).

Take log and differentiate (4) and (5) with respect to time, we obtain $\frac{\dot{Y}_{At}}{Y_{At}} = \alpha_A n$ and $\frac{\dot{Y}_{Mt}}{Y_{Mt}} = \alpha_M n$

in the asymptotic growth path. From (8), we have

$$g_Y^* \equiv \frac{\dot{Y}}{Y} = \frac{\omega_A Y_A^{\frac{\varepsilon-1}{\varepsilon}} \dot{Y}_A}{\omega_A Y_A^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_M^{\frac{\varepsilon-1}{\varepsilon}} Y_A} + \frac{\omega_M Y_M^{\frac{\varepsilon-1}{\varepsilon}} \dot{Y}_M}{\omega_A Y_A^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_M^{\frac{\varepsilon-1}{\varepsilon}} Y_M} = \begin{cases} \min \left\{ \frac{\dot{Y}_A}{Y_A}, \frac{\dot{Y}_M}{Y_M} \right\} & \text{if } \varepsilon < 1 \\ \max \left\{ \frac{\dot{Y}_A}{Y_A}, \frac{\dot{Y}_M}{Y_M} \right\} & \text{if } \varepsilon > 1 \end{cases}.$$

Apply $\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = \frac{\dot{Y}}{Y} - n$ and we get per capita final output growth rate.

Proposition 3

Proof: Use (26) and (27) to obtain $\frac{Y_{Mt}}{Y_{At}} = l_{Mt}^{\alpha_M} l_{At}^{-\alpha_A} k_{Mt}^{\beta_M} k_{At}^{-\beta_A} \tau_{Mt}^{\gamma_M} \tau_{At}^{-\gamma_A} \frac{M_t}{A_t} L_t^{\alpha_M - \alpha_A} K_t^{\beta_M - \beta_A} T^{\gamma_M - \gamma_A}$.

Plug it to (38) and we get

$$(A.3) \quad l_{Mt}^{-1} - 1 = \frac{\omega_A \alpha_A}{\omega_M \alpha_M} \left(l_{Mt}^{\alpha_M} l_{At}^{-\alpha_A} k_{Mt}^{\beta_M} k_{At}^{-\beta_A} \tau_{Mt}^{\gamma_M} \tau_{At}^{-\gamma_A} \frac{M_t}{A_t} L_t^{\alpha_M - \alpha_A} K_t^{\beta_M - \beta_A} T^{\gamma_M - \gamma_A} \right)^{\frac{1-\varepsilon}{\varepsilon}}.$$

Take log and differentiate (A.3) with respect to $\ln L_t$, we get

$$(A.4) \quad \frac{-1}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln L_t} = \left(\frac{1-\varepsilon}{\varepsilon} \right) \left[\alpha_M \frac{d \ln l_{Mt}}{d \ln L_t} + \alpha_A \frac{l_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln L_t} + \beta_M \frac{d \ln k_{Mt}}{d \ln L_t} + \beta_A \frac{k_{Mt}}{1-k_{Mt}} \cdot \frac{d \ln k_{Mt}}{d \ln L_t} + \gamma_M \frac{d \ln \tau_{Mt}}{d \ln L_t} + \gamma_A \frac{\tau_{Mt}}{1-\tau_{Mt}} \cdot \frac{d \ln \tau_{Mt}}{d \ln L_t} + (\alpha_M - \alpha_A) \right].$$

Also, take log and differentiate (39) and (40) with respect to $\ln L_t$, we get (42) and (43) respectively.

Plug (42) and (43) into (A.4), we get (41).

Proposition 4

Proof: Take log and differentiate (A.3) with respect to $\ln A_t$, we get

$$(A.5) \quad \frac{-1}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln A_t} = \left(\frac{1-\varepsilon}{\varepsilon} \right) \left[\alpha_M \frac{d \ln l_{Mt}}{d \ln A_t} + \alpha_A \frac{l_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln A_t} + \beta_M \frac{d \ln k_{Mt}}{d \ln A_t} + \beta_A \frac{k_{Mt}}{1-k_{Mt}} \cdot \frac{d \ln k_{Mt}}{d \ln A_t} + \gamma_M \frac{d \ln \tau_{Mt}}{d \ln A_t} + \gamma_A \frac{\tau_{Mt}}{1-\tau_{Mt}} \cdot \frac{d \ln \tau_{Mt}}{d \ln A_t} - 1 \right].$$

Also, take log and differentiate (39) and (40) with respect to $\ln A_t$, we get (45) and (46) respectively.

Plug (45) and (46) into (A.5), we get (44).

Proposition 5

Proof: Take log and differentiate (A.3) with respect to $\ln M_t$, we get

$$(A.6) \quad \frac{-1}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln M_t} = \left(\frac{1-\varepsilon}{\varepsilon} \right) \left[\alpha_M \frac{d \ln l_{Mt}}{d \ln M_t} + \alpha_A \frac{l_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln M_t} + \beta_M \frac{d \ln k_{Mt}}{d \ln M_t} + \beta_A \frac{k_{Mt}}{1-k_{Mt}} \cdot \frac{d \ln k_{Mt}}{d \ln M_t} + \gamma_M \frac{d \ln \tau_{Mt}}{d \ln M_t} + \gamma_A \frac{\tau_{Mt}}{1-\tau_{Mt}} \cdot \frac{d \ln \tau_{Mt}}{d \ln M_t} + 1 \right].$$

Also, take log and differentiate (39) and (40) with respect to $\ln M_t$, we get (48) and (49) respectively.

Plug (48) and (49) into (A.6), we get (47).

Proposition 6

Proof: Take log and differentiate (A.3) with respect to $\ln K_t$, we get

$$(A.7) \quad \frac{-1}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln K_t} = \left(\frac{1-\varepsilon}{\varepsilon} \right) \left[\alpha_M \frac{d \ln l_{Mt}}{d \ln K_t} + \alpha_A \frac{l_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln K_t} + \beta_M \frac{d \ln k_{Mt}}{d \ln K_t} + \beta_A \frac{k_{Mt}}{1-k_{Mt}} \cdot \frac{d \ln k_{Mt}}{d \ln K_t} + \gamma_M \frac{d \ln \tau_{Mt}}{d \ln K_t} + \gamma_A \frac{\tau_{Mt}}{1-\tau_{Mt}} \cdot \frac{d \ln \tau_{Mt}}{d \ln K_t} + (\beta_M - \beta_A) \right].$$

Also, take log and differentiate (39) and (40) with respect to $\ln K_t$, we get (51) and (52) respectively.

Plug (51) and (52) into (A.7), we get (50).

Proposition 7

Proof: Since the manufacturing sector is the asymptotically dominant sector, use the modified

form of (39) $k_{Mt} = \left[1 + \frac{\omega_A \beta_A}{\omega_M \beta_M} \left(\frac{Y_{Mt}}{Y_{At}} \right)^{\frac{1-\varepsilon}{\varepsilon}} \right]^{-1}$, with (31), we can verify that $\eta_t \equiv \omega_M^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{\beta_M}{\beta_A} \right) \left(\frac{1-k_{Mt}}{k_{Mt}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right] = \frac{Y_t}{Y_{Mt}}$. This implies $Y_t = \eta_t Y_{Mt}$.

Plug (27) into $Y_t = \eta_t Y_{Mt}$ to get

$$(A.8) \quad Y_t = \eta_t M_t L_t^{\alpha_M} K_t^{\beta_M} T^{\gamma_M} l_{Mt}^{\alpha_M} k_{Mt}^{\beta_M} \tau_{Mt}^{\gamma_M}.$$

By $\Phi_K = R_t$ and (36), $\Phi_K = \omega_M \beta_M \left(\frac{Y_t}{Y_{Mt}} \right)^{\frac{1}{\varepsilon}} \frac{Y_{Mt}}{K_{Mt}}$. Using (27) and $\frac{Y_t}{Y_{Mt}} = \eta_t$, we get

$$(A.9) \quad \Phi_K = \omega_M \beta_M \eta_t^{\frac{1}{\varepsilon}} M_t L_{Mt}^{\alpha_M} K_{Mt}^{\beta_M-1} T_{Mt}^{\gamma_M}.$$

Now take log and differentiate (56) and (57) with respect to time, we obtain

$$(A.10) \quad \frac{\dot{c}_t}{c_t} = \frac{\dot{\tilde{c}}_t}{\tilde{c}_t} - \frac{1}{1-\beta_M} g_M + \left(\frac{1-\alpha_M-\beta_M}{1-\beta_M} \right) n,$$

$$(A.11) \quad \frac{\dot{\chi}_t}{\chi_t} = \frac{1-\beta_M}{\alpha_M} \frac{\dot{K}_t}{K_t} - n - \frac{g_M}{\alpha_M}.$$

Plug (A.9) and (55) into (A.10) to obtain (58). Plug (A.8) and (54) into (A.11) to get (59).

Manipulate (36) to get $k_{Mt} = \left[1 + \frac{\omega_A \beta_A}{\omega_M \beta_M} \left(\frac{Y_{Mt}}{Y_{At}} \right)^{\frac{1-\varepsilon}{\varepsilon}} \right]^{-1}$. Take log and differentiate the expression with respect to time to get

$$(A.12) \quad \frac{\dot{k}_{Mt}}{k_{Mt}} = (1 - k_{Mt}) \left(\frac{\varepsilon-1}{\varepsilon} \right) \left(\frac{\dot{Y}_{Mt}}{Y_{Mt}} - \frac{\dot{Y}_{At}}{Y_{At}} \right).$$

Take log and differentiate (26), (27) with respect to time and plug into (A.12), we get

$$\begin{aligned} \frac{\dot{k}_{Mt}}{k_{Mt}(1-k_{Mt})} \left(\frac{\varepsilon}{\varepsilon-1} \right) &= \left(\frac{\dot{M}_t}{M_t} - \frac{\dot{A}_t}{A_t} \right) + \beta_M \left(\frac{\frac{d(k_{Mt} K_t)}{dt}}{k_{Mt} K_t} \right) - \beta_A \left(\frac{\frac{d(k_{At} K_t)}{dt}}{k_{At} K_t} \right) + \alpha_M \left(\frac{\frac{d(l_{Mt} L_t)}{dt}}{l_{Mt} L_t} \right) - \alpha_A \left(\frac{\frac{d(l_{At} L_t)}{dt}}{l_{At} L_t} \right) + \\ &\gamma_M \left(\frac{\frac{d(\tau_{Mt} T)}{dt}}{\tau_{Mt} T} \right) - \gamma_A \left(\frac{\frac{d(\tau_{At} T)}{dt}}{\tau_{At} T} \right). \end{aligned}$$

Apply product rule and manipulate the above equation to obtain

$$(A.13) \quad \begin{aligned} \frac{\dot{k}_{Mt}}{k_{Mt}(1-k_{Mt})} \left(\frac{\varepsilon}{\varepsilon-1} \right) &= (g_M - g_A) + (\beta_M - \beta_A) \left(\frac{\dot{k}_{Mt}}{k_{Mt}} + \frac{\dot{K}_t}{K_t} \right) + \beta_A \left(\frac{\dot{k}_{Mt}}{k_{Mt}(1-k_{Mt})} \right) + \\ &(\alpha_M - \alpha_A) \left(\frac{\dot{l}_{Mt}}{l_{Mt}} + \frac{\dot{L}_t}{L_t} \right) + \alpha_A \left(\frac{\dot{l}_{Mt}}{l_{Mt}(1-l_{Mt})} \right) + (\gamma_M - \gamma_A) \left(\frac{\dot{\tau}_{Mt}}{\tau_{Mt}} \right) + \gamma_A \left(\frac{\dot{\tau}_{Mt}}{\tau_{Mt}(1-\tau_{Mt})} \right). \end{aligned}$$

Take log and differentiate (39), (40) with respect to time to get (63) and (64) respectively.

Plug (63), (64) and (A.11) into (A.13), we obtain (60).

To ensure the transversality condition is satisfied, we require

$$(A.14) \quad \lim_{t \rightarrow \infty} \mu_t K_t = 0,$$

where μ_t is the costate variable in the Hamiltonian $H(\tilde{c}_t, K_t, \mu_t) \equiv e^{-(\rho-n)t} \left(\frac{\tilde{c}_t^{1-\theta} - 1}{1-\theta} \right) + \mu_t [\Phi(K_t, t) - \delta K_t - e^{nt} L_0 \tilde{c}_t]$.

Maximum principle requires $H_{\tilde{c}_t} = 0$, which implies $e^{-(\rho-n)t} \tilde{c}_t^{-\theta} - \mu_t e^{nt} L_0 = 0$. Using (56),

$$\mu_t = \frac{e^{-\rho t}}{L_0} c_t^{-\theta} M_t^{\frac{-\theta}{1-\beta_M}} L_t^{\left(\frac{1-\alpha_M-\beta_M}{1-\beta_M} \right) \theta}.$$

Plugging it into (A.14), and note from (57) that $K_t = L_t^{\frac{\alpha_M}{1-\beta_M}} M_t^{\frac{1}{1-\beta_M}} \chi_t^{\frac{\alpha_M}{1-\beta_M}}$, with some algebra we get (62).

Proposition 9 will state that $c_t \rightarrow c^*$ and $\chi_t \rightarrow \chi^*$ asymptotically, and the transversality is equivalent to (A4).

Proposition 8

Proof: From (A.10) CGP requires $\frac{\dot{c}_t}{c_t}$ to be a constant. By (58) it implies $\eta_t^{\frac{1}{\varepsilon}} \chi_t^{-\alpha_M} l_{Mt}^{\alpha_M} k_{Mt}^{\beta_M-1} \tau_{Mt}^{\gamma_M}$

is constant or

$$(A.15) \quad \frac{1}{\varepsilon} \frac{\dot{\eta}_t}{\eta_t} - \alpha_M \frac{\dot{\chi}_t}{\chi_t} + \alpha_M \frac{\dot{l}_{Mt}}{l_{Mt}} + (\beta_M - 1) \frac{\dot{k}_{Mt}}{k_{Mt}} + \gamma_M \frac{\dot{\tau}_{Mt}}{\tau_{Mt}} = 0.$$

On the other hand, take log and differentiate (61) with respect to time to get

$$(A.16) \quad \frac{\dot{\eta}_t}{\eta_t} = \frac{\left(\frac{\varepsilon}{\varepsilon-1} \right) \left(\frac{\beta_M}{\beta_A} \right) \left(\frac{-1}{k_{Mt} k_{Mt}} \right)}{1 + \frac{\beta_M}{\beta_A} \left(\frac{1-k_{Mt}}{k_{Mt}} \right)} = \frac{-\left(\frac{\varepsilon}{\varepsilon-1} \right) \beta_M \frac{k_{Mt}}{k_{Mt}}}{\beta_A k_{Mt} + \beta_M (1-k_{Mt})}.$$

Plug (A.15), (A.16), (63) and (64) into (60) to get (65), where

$$G(k_{Mt}) \equiv \frac{\left(\frac{1-\beta_M}{\beta_M-\beta_A}\right)^{(1-k_{Mt})}}{\frac{-\left(\frac{1}{\varepsilon-1}\right)\beta_M(1-k_{Mt})}{\beta_A k_{Mt} + \beta_M(1-k_{Mt})} + \alpha_M(1-l_{Mt}) + \gamma_M(1-\tau_{Mt}) - \left(\frac{1-\beta_M}{\beta_M-\beta_A}\right)\left(\frac{\varepsilon}{\varepsilon-1}\right) + \left(\frac{1-\beta_M}{\beta_M-\beta_A}\right)[(\alpha_M-\alpha_A)(1-l_{Mt}) + (\gamma_M-\gamma_A)(1-\tau_{Mt}) + \alpha_A + \beta_A + \gamma_A]}.$$

Observe that $G(1) = 0$, and $G(k_{Mt}) > 0 \ \forall k_{Mt} \in [0, 1)$, given (A1), (A2) hold. Hence when (A3) is also satisfied, by (65) $\dot{k}_{Mt} > 0$ and $k_{Mt} \rightarrow 1$ as $t \rightarrow \infty$. By (39) and (40), $l_{Mt} \rightarrow 1$ and $\tau_{Mt} \rightarrow 1$ as $t \rightarrow \infty$ too.

Proposition 9

Proof: We first solve for the steady state allocation in CGP. From proposition 8, given (A1)-(A4), $k_{Mt} \rightarrow k_M^* = 1$, $l_{Mt} \rightarrow l_M^* = 1$ and $\tau_{Mt} \rightarrow \tau_M^* = 1$ as $t \rightarrow \infty$. By (61) $\eta_t \rightarrow \omega_M \frac{\varepsilon}{\varepsilon-1}$. From (A.15), $\chi_t \rightarrow \chi^*$ also exists. We can solve for χ^* by (58), c^* by (59), g_c^* by (A.10), g_K^* by (A.11).

Since manufacturing sector is the asymptotically dominant sector, by (28)-(30) and $k_M^* = l_M^* = \tau_M^* = 1$, we have $g_{K_M}^* = g_K^*$, $g_{L_M}^* = g_L^* = n$ and $g_{T_M}^* = g_T^* = 0$.

$$\text{By (27), } g_{Y_M}^* \equiv \frac{\dot{Y}_M}{Y_M} = \frac{\dot{M}}{M} + \alpha_M \frac{\dot{L}}{L} + \beta_M \frac{\dot{K}}{K} = g_M + \alpha_M n + \beta_M \left(\frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} \right).$$

$$\text{For the agricultural sector, by (26), } g_{Y_A}^* \equiv \frac{\dot{Y}_A}{Y_A} = \frac{\dot{A}}{A} + \alpha_A \frac{\dot{L}}{L} + \beta_A \frac{\dot{K}}{K} = g_A + \alpha_A n + \beta_A \left(\frac{\alpha_M}{1-\beta_M} n + \frac{g_M}{1-\beta_M} \right).$$

$$\text{Note } g_{Y_A}^* - g_{Y_M}^* = \left[g_A - \left(\frac{1-\beta_A}{1-\beta_M} \right) g_M \right] + \left[\alpha_A - \left(\frac{1-\beta_A}{1-\beta_M} \right) \alpha_M \right] n > 0 \text{ by (A3).}$$

$$\text{From the second equality in (35) we have } \frac{1}{\varepsilon} \frac{\dot{Y}}{Y} + \left(1 - \frac{1}{\varepsilon} \right) \frac{\dot{Y}_A}{Y_A} - \frac{\dot{L}_A}{L_A} = \frac{1}{\varepsilon} \frac{\dot{Y}}{Y} + \left(1 - \frac{1}{\varepsilon} \right) \frac{\dot{Y}_M}{Y_M} - \frac{\dot{L}_M}{L_M}, \text{ which}$$

$$\text{implies } g_{L_A}^* \equiv \frac{\dot{L}_A}{L_A} = \left(1 - \frac{1}{\varepsilon} \right) (g_{Y_A}^* - g_{Y_M}^*) + g_{L_M}^* < g_{L_M}^*, \text{ given (A1).}$$

$$\text{Similarly, from the second equality in (36) we have } \left(1 - \frac{1}{\varepsilon} \right) \frac{\dot{Y}_A}{Y_A} - \frac{\dot{K}_A}{K_A} = \left(1 - \frac{1}{\varepsilon} \right) \frac{\dot{Y}_M}{Y_M} - \frac{\dot{K}_M}{K_M}, \text{ which}$$

$$\text{implies } g_{K_A}^* \equiv \frac{\dot{K}_A}{K_A} = \left(1 - \frac{1}{\varepsilon} \right) (g_{Y_A}^* - g_{Y_M}^*) + g_{K_M}^* < g_{K_M}^*, \text{ given (A1). And from the second}$$

$$\text{equality in (37) we have } \left(1 - \frac{1}{\varepsilon} \right) \frac{\dot{Y}_A}{Y_A} - \frac{\dot{T}_A}{T_A} = \left(1 - \frac{1}{\varepsilon} \right) \frac{\dot{Y}_M}{Y_M} - \frac{\dot{T}_M}{T_M}, \text{ which implies } g_{T_A}^* \equiv \frac{\dot{T}_A}{T_A} =$$

$$\left(1 - \frac{1}{\varepsilon} \right) (g_{Y_A}^* - g_{Y_M}^*) + g_{T_M}^* < g_{T_M}^*, \text{ given (A1).}$$

$$\text{From the proof in proposition 2, } g_Y^* \equiv \frac{\dot{Y}}{Y} = \min \left\{ \frac{\dot{Y}_A}{Y_A}, \frac{\dot{Y}_M}{Y_M} \right\} = g_{Y_M}^*, \text{ given (A1). Hence } g_y^* = g_Y^* -$$

$$n = \frac{g_M}{1-\beta_M} - \left(\frac{1-\alpha_M-\beta_M}{1-\beta_M} \right) n.$$

(A4) plus $c_t \rightarrow c^*$ and $\chi_t \rightarrow \chi^*$ ensures the transversality condition (62) is satisfied. Together with household's period utility function $\frac{\tilde{c}_t^{1-\theta}-1}{1-\theta}$ being strictly concave in \tilde{c}_t , (58)-(60) is the unique dynamic equilibrium characterizing the social planner's solution to (33). (Acemoglu 2009, Thm. 7.8).

To prove that the dynamic equilibrium converges to the CGP, we study the saddle-path property of the linearized dynamic system around the asymptotic state (CGP steady state). We rewrite the system (58)-(60) as

$$(A.17) \quad \dot{X}_t = f(X_t), \text{ where } X_t \equiv \begin{pmatrix} c_t \\ \chi_t \\ k_{Mt} \end{pmatrix}.$$

Define $z_t \equiv X_t - X^*$. We linearize (A.17) around the asymptotic state X^* to get

$$(A.18) \quad \dot{z}_t = J(X^*)z_t, \text{ where } J(X^*) = \begin{pmatrix} 0 & a_{c\chi} & a_{ck_M} \\ -\frac{1-\beta_M}{\alpha_M}(\chi^*)^{1-\frac{\alpha_M}{1-\beta_M}} & a_{\chi\chi} & a_{\chi k_M} \\ 0 & 0 & a_{k_M k_M} \end{pmatrix}$$

is value of the Jacobian matrix of the system (A.17) at the asymptotic state X^* .

- From (58), $a_{c\chi} = \frac{-\alpha_M}{\theta} \left[\omega_M \beta_M (\eta^*)^{\frac{1}{\varepsilon}} (\chi^*)^{-\alpha_M-1} T^{\gamma_M} \right] c^* < 0$.
- From (60), $a_{k_M k_M} = \frac{g_A - \left(\frac{1-\beta_A}{1-\beta_M} \right) g_M - \left[\alpha_M - \alpha_A + \frac{\alpha_M(\beta_M - \beta_A)}{1-\beta_M} \right] n}{\frac{\varepsilon}{\varepsilon-1} - (\alpha_A + \beta_A + \gamma_A)} < 0$, given (A1)-(A3).

This implies that $|J(X^*)| = \frac{1-\beta_M}{\alpha_M} (\chi^*)^{1-\frac{\alpha_M}{1-\beta_M}} a_{c\chi} a_{k_M k_M} > 0$ and all eigenvalues of $J(X^*)$ have

non-zero real parts.⁵⁸ Hence the asymptotic state X^* is hyperbolic. By the Grobman-Hartman Theorem, the dynamics of the nonlinear system (A.17) in the neighborhood of X^* is qualitatively the same as the dynamics of the linearized system (A.18). (Acemoglu 2009, Thm. B.7).

Next we set up the characteristics equation for the Jacobian matrix evaluated at the asymptotic

$$\text{state: } |J(X^*) - vI| = 0 \Rightarrow \begin{vmatrix} -v & a_{c\chi} & a_{ck_M} \\ -\frac{1-\beta_M}{\alpha_M}(\chi^*)^{1-\frac{\alpha_M}{1-\beta_M}} & a_{\chi\chi} - v & a_{\chi k_M} \\ 0 & 0 & a_{k_M k_M} - v \end{vmatrix} = 0.$$

$$\Rightarrow (a_{k_M k_M} - v) \left[v^2 - a_{\chi\chi} v + \frac{1-\beta_M}{\alpha_M} (\chi^*)^{1-\frac{\alpha_M}{1-\beta_M}} \cdot a_{c\chi} \right] = 0.$$

Since $a_{k_M k_M}, a_{c\chi} < 0$, the above characteristic equation has two negative roots and one positive root, which implies that the asymptotic state is saddle-path stable. That means, there exists a unique two-dimensional manifold of solutions to the dynamic system (58)-(60) converging to the CGP steady state.

Appendix 2: Land Expansion Effect and Structural Transformation in the United States, AD1790-AD1870

Proposition A.1 states how a one-time increase in land supply in an economy affects sectoral shares in the unified model in section 5.

Proposition A.1 (Land expansion effect): In a competitive equilibrium,
(A.19)

⁵⁸ We directly assume $c^*, \chi^* > 0$. Note that assuming $\chi^* > 0$ also assures sustainable per capita income growth.

$$\frac{d \ln l_{Mt}}{d \ln T} = - \frac{(1-\varepsilon)(\gamma_M - \gamma_A)(1-l_{Mt})}{\varepsilon + (1-\varepsilon)[\alpha_M(1-l_{Mt}) + \alpha_A l_{Mt} + \beta_M(1-k_{Mt}) + \beta_A k_{Mt} + \gamma_M(1-\tau_{Mt}) + \gamma_A \tau_{Mt}]} \begin{cases} < 0 & \text{if } \varepsilon < 1 \text{ and } \gamma_M > \gamma_A \\ > 0 & \text{if } \varepsilon < 1 \text{ and } \gamma_M < \gamma_A \end{cases}$$

$$(A.20) \quad \frac{d \ln k_{Mt}}{d \ln T} = \frac{1-k_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln T},$$

$$(A.21) \quad \frac{d \ln \tau_{Mt}}{d \ln T} = \frac{1-\tau_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln T}.$$

Proof: Take log and differentiate (A.3) with respect to $\ln T$, we get

$$(A.22) \quad \frac{-1}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln T} = \left(\frac{1-\varepsilon}{\varepsilon} \right) \left[\alpha_M \frac{d \ln l_{Mt}}{d \ln T} + \alpha_A \frac{l_{Mt}}{1-l_{Mt}} \cdot \frac{d \ln l_{Mt}}{d \ln T} + \beta_M \frac{d \ln k_{Mt}}{d \ln T} + \beta_A \frac{k_{Mt}}{1-k_{Mt}} \cdot \frac{d \ln k_{Mt}}{d \ln T} + \gamma_M \frac{d \ln \tau_{Mt}}{d \ln T} + \gamma_A \frac{\tau_{Mt}}{1-\tau_{Mt}} \cdot \frac{d \ln \tau_{Mt}}{d \ln T} + (\gamma_M - \gamma_A) \right].$$

Also, take log and differentiate (39) and (40) with respect to $\ln T$, we get (A.20) and (A.21) respectively.

Plug (A.20) and (A.21) into (A.22), we get (A.19).

Similar to population growth effect, technology growth effects and capital deepening effect (propositions 3-6), land expansion effect operates through the relative price effect. *Ceteris paribus*, if $\varepsilon < 1$, land expansion generates a more than proportionate relative price drop (compared to the relative marginal product of land rise) in the sector with higher land intensity. Land use shifts out of this sector until the land rental parity condition (37) is restored. Due to input complementarity, capital and labor also move in the same direction.

To apply proposition A.1, we need to find historical episodes where a country expanded its territories over thinly populated areas. The United States during AD1790-AD1870 fits this criterion. Turner (1976[1920], 3) stated that, up to the late-nineteenth century,

"[t]he American frontier is sharply distinguished from the European frontier—a fortified boundary line running through dense populations. The most significant thing about the American frontier is, that it lies at the hither edge of free land. In the census reports it is treated as the margin of that settlement which has a density of two or more to the square mile."

Table A.1 shows the United States territorial expansion (in terms of land and water area) during AD1790-AD1870. The United States had increased its land and water area fourfold during the eighty years. Since the population density was thin in the newly acquired land, we might treat these territorial expansions as exogenous land supply increases in the United States economy. Applying proposition A.1, given $\varepsilon < 1$, our theory predicts that the United States territorial expansion would shift labor, capital and land use from the agricultural to the manufacturing sector – that is, land expansion would foster industrialization in the United States.

INSERT TABLE A.1 HERE

Our theoretic prediction is consistent with the fact that agricultural labor share in the United States declined within this time frame. Kuznets (1966, 107) documented that agricultural share in total

labor force fell from 68% in AD1840 to 51% in AD1870, while Clark (1960, 520) noted that the share of labor force allocated to agriculture, fishing and forestry declined from 72.0% in AD1820 to 50.8% in AD1870.

Appendix 3: Other Proofs

A. Deriving the price indices (9)-(11)

Consider the choice problem of the final output producer. For whatever final output level Y_t the producer decides on, it is always optimal to purchase the combination of agricultural and manufacturing goods that minimize the cost of achieving the level Y_t , that is:

$$(A.23) \quad \min_{Y_{At}, Y_{Mt}} P_{At} Y_{At} + P_{Mt} Y_{Mt} \quad \text{subject to} \quad \left(\omega_A Y_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_{Mt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \geq Y_t .$$

We set up the Lagrangian for the problem (A.23):

$$(A.24) \quad \Pi_t = P_{At} Y_{At} + P_{Mt} Y_{Mt} + F_t \left[Y_t - \left(\omega_A Y_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_{Mt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \right] ,$$

where the Lagrangian multiplier F_t shows the shadow price of Y_t , that is, the price of final output at time t .

First order conditions with respect to Y_{At} and Y_{Mt} yields:

$$(A.25) \quad Y_{At} = \left(\frac{P_{At}}{F_t} \right)^{-\varepsilon} \omega_A^\varepsilon Y_t ,$$

$$(A.26) \quad Y_{Mt} = \left(\frac{P_{Mt}}{F_t} \right)^{-\varepsilon} \omega_M^\varepsilon Y_t .$$

Plug (A.25) and (A.26) into the definition of Y_t (equation (8) or (31)), solving for F_t ,

$$(A.27) \quad F_t = \left(\omega_A^\varepsilon P_{At}^{1-\varepsilon} + \omega_M^\varepsilon P_{Mt}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} .$$

By normalizing $F_t \equiv 1$ in (A.27), (A.25), (A.26), we obtain (9)-(11).

We note that the consumption composite price P_t always equals final output price F_t . To see this, consider the choice problem of the representative household in section 5.1.⁵⁹ For whatever consumption composite \tilde{c}_t the household chooses, it is always optimal to purchase the combination of agricultural and manufacturing goods that minimizes the cost of achieving the level \tilde{c}_t , that is:

$$(A.28) \quad \min_{\tilde{Y}_{At}, \tilde{Y}_{Mt}} P_{At} \tilde{Y}_{At} + P_{Mt} \tilde{Y}_{Mt} \\ \text{subject to} \quad \left(\omega_A \tilde{Y}_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M \tilde{Y}_{Mt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} - \frac{\dot{K}_t}{L_t} - \frac{\delta K_t}{L_t} \geq \tilde{c}_t , \quad \text{given } L_t, K_t > 0, \dot{K}_t .$$

We set up the Lagrangian for the problem (A.28):

$$(A.29) \quad \Pi'_t = P_{At} \tilde{Y}_{At} + P_{Mt} \tilde{Y}_{Mt} + P_t \left[\tilde{c}_t - \left(\omega_A \tilde{Y}_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M \tilde{Y}_{Mt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\dot{K}_t}{L_t} + \frac{\delta K_t}{L_t} \right] ,$$

where P_t is the shadow price of \tilde{c}_t , which is in the same form as in (A.23). By similar

⁵⁹ For section 4.1, ignore the terms involving K_t and \dot{K}_t .

procedures as in (A.24)-(A.27), we get $P_t = (\omega_A^\varepsilon P_{At}^{1-\varepsilon} + \omega_M^\varepsilon P_{Mt}^{1-\varepsilon})^{\frac{1}{1-\varepsilon}} = F_t = 1$ for all t .

B. Deriving the economy-wide resource constraint (12)

First, consider the choice problem faced by the agricultural producer:

$$(A.30) \quad \max_{L_{At}, T_{At}} P_{At} Y_{At} - W_t L_{At} - \Omega_t T_{At} \quad \text{subject to} \quad (4).$$

First order conditions implies:

$$(A.31) \quad W_t L_{At} = \alpha_A P_{At} Y_{At},$$

$$(A.32) \quad \Omega_t T_{At} = \gamma_A P_{At} Y_{At}.$$

Similarly, consider the choice problem faced by the manufacturing producer:

$$(A.33) \quad \max_{L_{Mt}, T_{Mt}} P_{Mt} Y_{Mt} - W_t L_{Mt} - \Omega_t T_{Mt} \quad \text{subject to} \quad (5).$$

First order conditions implies:

$$(A.34) \quad W_t L_{Mt} = \alpha_M P_{Mt} Y_{Mt},$$

$$(A.35) \quad \Omega_t T_{Mt} = \gamma_M P_{Mt} Y_{Mt}.$$

Now we multiple both sides of (2) by L_t and apply $P_t = 1$, (6)-(7) to get:

$$(A.36) \quad L_t \tilde{c}_t = W_t (L_{At} + L_{Mt}) + \Omega_t (T_{At} + T_{Mt}).$$

Apply (A.31)-(A.32) and (A.34)-(A.35) to (A.36). Using $\alpha_A + \gamma_A = 1$ and $\alpha_M + \gamma_M = 1$ to get:

$$(A.37) \quad L_t \tilde{c}_t = P_{At} Y_{At} + P_{Mt} Y_{Mt}.$$

Note that

$$(A.38) \quad P_{At} Y_{At} + P_{Mt} Y_{Mt} = \left(\omega_A Y_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_{Mt}^{\frac{\varepsilon-1}{\varepsilon}} \right) Y_t^{\frac{1}{\varepsilon}} = Y_t^{\frac{\varepsilon-1}{\varepsilon}} Y_t^{\frac{1}{\varepsilon}} = Y_t,$$

where the first equality comes from (10)-(11), and the second equality follows from the definition of Y_t (equation (8)). Plug (A.38) into (A.37) and we obtain (12).

C. Deriving the economy-wide resource constraint (32)

First, consider the choice problem faced by the agricultural producer:

$$(A.39) \quad \max_{L_{At}, K_{At}, T_{At}} P_{At} Y_{At} - W_t L_{At} - R_t K_{At} - \Omega_t T_{At} \quad \text{subject to} \quad (26).$$

First order conditions implies:

$$(A.40) \quad W_t L_{At} = \alpha_A P_{At} Y_{At},$$

$$(A.41) \quad R_t K_{At} = \beta_A P_{At} Y_{At},$$

$$(A.42) \quad \Omega_t T_{At} = \gamma_A P_{At} Y_{At}.$$

Similarly, consider the choice problem faced by the manufacturing producer:

$$(A.43) \quad \max_{L_{Mt}, K_{Mt}, T_{Mt}} P_{Mt} Y_{Mt} - W_t L_{Mt} - R_t K_{Mt} - \Omega_t T_{Mt} \quad \text{subject to} \quad (27).$$

First order conditions implies:

$$(A.44) \quad W_t L_{Mt} = \alpha_M P_{Mt} Y_{Mt},$$

$$(A.45) \quad R_t K_{Mt} = \beta_M P_{Mt} Y_{Mt},$$

$$(A.46) \quad \Omega_t T_{Mt} = \gamma_M P_{Mt} Y_{Mt}.$$

Now we multiple both sides of (24) by L_t and apply $P_t = 1$, $r_t = R_t - \delta$, (28)-(30) to get:

$$(A.47) \quad \dot{K}_t = W_t (L_{At} + L_{Mt}) + R_t (K_{At} + K_{Mt}) - \delta K_t + \Omega_t (T_{At} + T_{Mt}) - L_t \tilde{c}_t.$$

Apply (A.40)-(A.42) and (A.44)-(A.46) to (A.47). Using $\alpha_A + \beta_A + \gamma_A = 1$ and $\alpha_M + \beta_M + \gamma_M = 1$ to get:

$$(A.48) \quad \dot{K}_t = P_{At}Y_{At} + P_{Mt}Y_{Mt} - \delta K_t - L_t\tilde{c}_t .$$

Note that

$$(A.49) \quad P_{At}Y_{At} + P_{Mt}Y_{Mt} = \left(\omega_A Y_{At}^{\frac{\varepsilon-1}{\varepsilon}} + \omega_M Y_{Mt}^{\frac{\varepsilon-1}{\varepsilon}} \right) Y_t^{\frac{1}{\varepsilon}} = Y_t^{\frac{\varepsilon-1}{\varepsilon}} Y_t^{\frac{1}{\varepsilon}} = Y_t ,$$

where the first equality comes from (10)-(11), and the second equality follows from the definition of Y_t (equation (31)). Plug (A.49) into (A.48) and we obtain (32).

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TABLE 1
*Annual agricultural productivity growth rate,
England, AD1525-AD1795*

Period	Annual agricultural productivity growth rate (%)
1525-1605	-0.06
1605-1745	0.15
1745-1795	-0.12

Source: Clark (2002), Table 7.

TABLE 2
*Benchmark parameter values to simulate structural transformation in pre-industrial England,
AD1521-AD1745*

	Interpretation	Value	Comments/Observations
<i>Parameters</i>			
α_A	Labor intensity in agricultural sector	0.4	Vollrath (2011)
γ_A	Land intensity in agricultural sector	0.6	Vollrath (2011)
α_M	Labor intensity in manufacturing sector	1	Yang and Zhu (2013)
γ_M	Land intensity in manufacturing sector	0.01	Yang and Zhu (2013)
ε	Elasticity of substitution	0.5	Buera and Kaboski (2009)
<i>Initial values</i>			
l_{A0}	Initial agricultural labor share	0.581	Match Broadberry et al. (2013)'s AD1522 estimate
t_{A0}	Initial agricultural land share	0.95	> AD1961 agricultural land share
<i>Annual growth rates</i>			
g_A	Agricultural technology growth rate	1521-1605: -0.0006 1606-1745: 0.0015	Clark (2002)
g_M	Manufacturing technology growth rate	0%	Assumption
n	Population growth rate	1521-1550: 0% 1551-1605: 0.0064 1606-1660: 0.0036 1661-1745: 0.0010	Population estimates from Mitchell (1988) and Pamuk (2007)

TABLE 3
*Baseline parameter values to simulate structural transformation in the modern United States,
AD1980-AD2100*

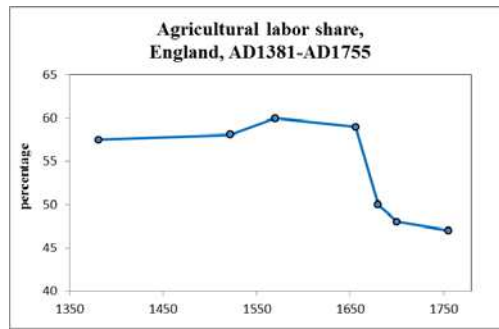
	Interpretation	Value	Comments/Observations
<i>Parameters</i>			
α_A	Labor intensity in agricultural sector	0.6	Gollin et al. (2007)
β_A	Capital intensity in agricultural sector	0.1	Gollin et al. (2007)
γ_A	Land intensity in agricultural sector	0.3	Gollin et al. (2007)
α_M	Labor intensity in manufacturing sector	0.5	Gollin et al. (2007)
β_M	Capital intensity in manufacturing sector	0.5	Gollin et al. (2007)
γ_M	Land intensity in manufacturing sector	0.01	Gollin et al. (2007)
ε	Elasticity of substitution	0.5	Buera and Kaboski (2009)
<i>Initial values</i>			
k_{A0}	Initial agricultural capital share	0.030868	U.S. Bureau of Economic Analysis (2016), proportion of fixed assets (chain-type quantity indexes) held by agricultural, forestry, fishing and hunting sectors in AD1980
l_{A0}	Initial agricultural labor share	0.036000	The World Bank (2016), % of total employment in agriculture in AD1980
t_{A0}	Initial agricultural land share	0.46748	The World Bank (2016), % of land area in agriculture in AD1980
<i>Annual growth rates</i>			
g_A	Agricultural technology growth rate	0.0220	U.S. Department of Agriculture (2016), annualized farm TFP growth rate, AD1980-AD2002
g_M	Manufacturing technology growth rate	0.0075	U.S. Bureau of Labor Statistics (2016), annualized private nonfarm business sectors multifactor productivity growth rate, AD1980-AD2002
n	Population growth rate	0.0148	U.S. Bureau of Economic Analysis (2016), annualized full-time and part-time employees growth rate, AD1980-AD2002

TABLE A.1
Territorial expansion and land and water area
of the United States, in square miles, AD1790-AD1870

<i>Year</i>	<i>Territorial expansion</i>	<i>Gross Area (land and water)</i>
1790	United States territory in AD1790	888,685
1803	Louisiana Purchase	827,192
1819	Treaty with Spain	72,003
1845	Texas	390,143
1846	Oregon	285,580
1848	Mexican Cession	529,017
1853	Gadsden Purchase	29,640
1867	Alaska Purchase	586,412
	United States territory in AD1870	3,608,672

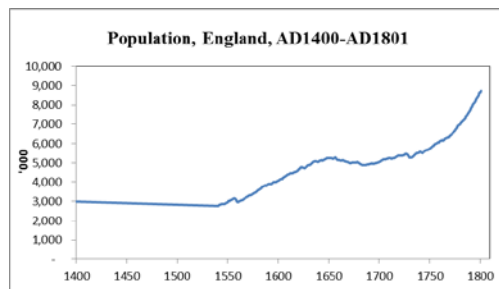
Source: U.S. Bureau of the Census (1975). Historical Statistics of the United States, Series J 1-2.

FIGURE 1



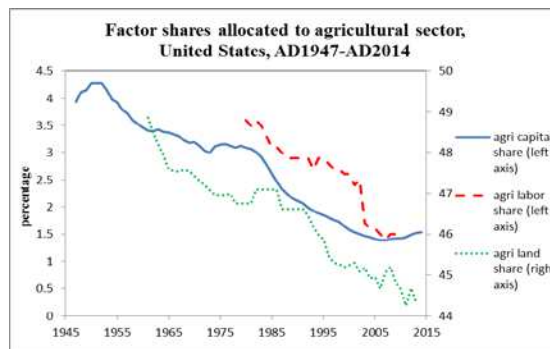
Note: Agricultural labor share, England, AD1381-AD1755. Source: Clark (2013) Table 2 and section 7 for AD1381-AD1660 estimates. Clark (2010) for AD1680-AD1755 estimates. Broadberry et al. (2013) Table 9 for AD1522 estimate.

FIGURE 2



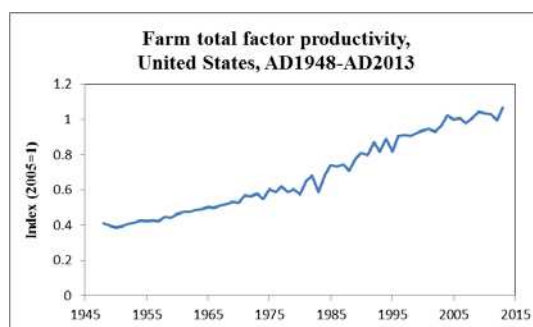
Note: Population, England, AD1400-AD1801. Source: Mitchell (1988), Table I-1 for AD1541-AD1800 England (excluding Monmouthshire) estimates, Pamuk (2007) Table 1 for AD1400 England and Wales estimate.

FIGURE 3



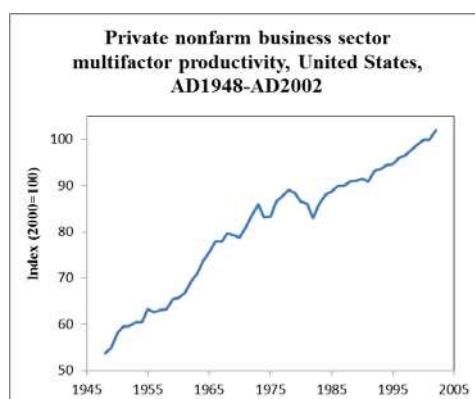
Note: Factor shares allocated to agricultural sector, United States, AD1947-AD2014. Solid (blue) line: Agricultural capital share. Dashed (red) line: Agricultural labor share. Dotted (green) line: Agricultural land share. Source: capital share calculated as proportion of private fixed assets (chained-type quantity indexes) held by agricultural, forestry, fishing and hunting sectors, using NIPA Tables (301ESI Ann) provided by U.S. Bureau of Economic Analysis (2016); labor and land shares are respectively employment in agriculture (% of total employment), agricultural land (% of land area), provided by the World Bank (2016).

FIGURE 4



Note: Farm total factor productivity, United States (2005=1), AD1948-AD2013. Source: Economic Research Service, U.S. Department of Agriculture (2016).

FIGURE 5



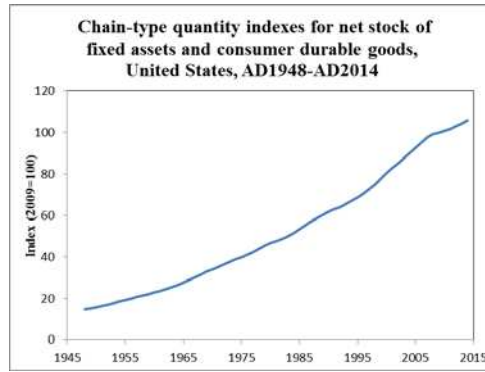
Note: Private nonfarm business sector multifactor productivity (SIC measures), United States (2000=100), AD1948-AD2002. Source: U.S. Bureau of Labor Statistics (2016).

FIGURE 6



Note: Full-time and part-time employees, United States, AD1969-AD2014. Source: NIPA Tables (60400B Ann, 60400C Ann, 60400D Ann) provided by U.S. Bureau of Economic Analysis (2016).

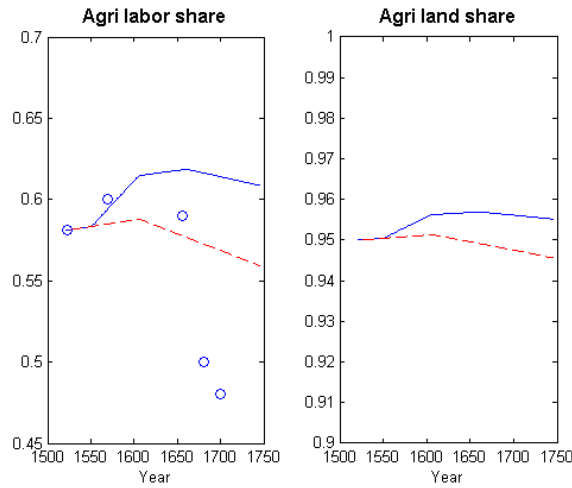
FIGURE 7



Note: Chain-type quantity indexes for net stock of fixed assets and consumer durable goods, United States (2009=100), AD1948-AD2014. Source: NIPA Tables (102 Ann) provided by U.S. Bureau of Economic Analysis (2016).

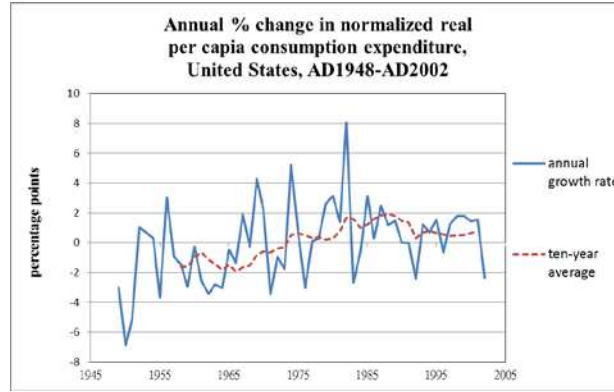
FIGURE 8

*Simulated agricultural labor and land shares,
England, AD1521-AD1745*



Note: Solid (blue) line: simulated English economy. Dashed (red) lines: the counterfactual economy, $n = 0 \forall t$, otherwise benchmark parameters from Table 2. Dots (blue): agricultural labor share estimates from Clark (2010, 2013) (Figure 1). The left and right panels show respectively the simulated agricultural labor and land shares.

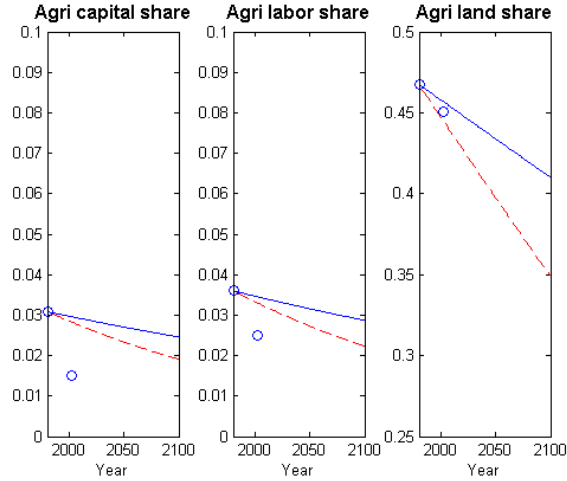
FIGURE 9



Note: Solid (blue) line: Annual % change in normalized real per capita consumption expenditure, United States, AD1948-AD2002. Dashed (red) line: ten-year average series. Source: Federal Reserve Bank of St. Louis (2016), Real personal consumption expenditures per capita, Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate.

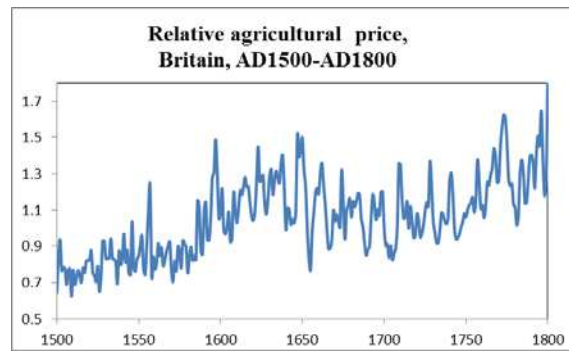
FIGURE 10

Simulated agricultural capital, labor and land shares, United States, AD1980-AD2100



Note: Solid (blue) line: simulated United States economy. Dashed (red) lines: the counterfactual economy, $n = 0$ otherwise baseline parameters from Table 3. Dots (blue): agricultural capital share is calculated as proportion of fixed assets held by agricultural, forestry, fishing and hunting sectors in AD1980 and AD2002, using BEA (2016) data; agricultural labor share is calculated as % of total employment in agriculture in AD1980 and AD2002, from the World Bank (2016); agricultural land share is calculated as % of land area in agriculture in AD1980 and AD2002, from the World Bank (2016) (Figure 3). The left, middle and right panels show respectively the simulated agricultural capital, labor and land shares.

FIGURE 11



Note: Relative agricultural price, Britain, AD1500-AD1800. Source: Broadberry et al. (2011)
Agriculture price index divided by Industry price index.

FIGURE 12A

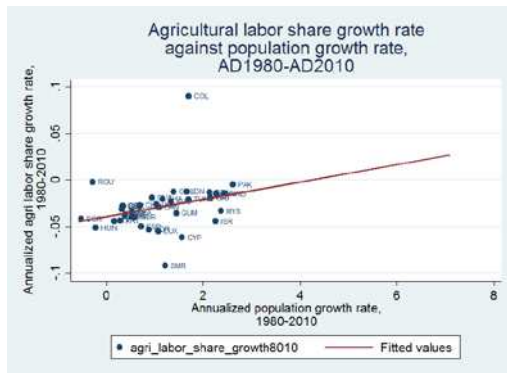
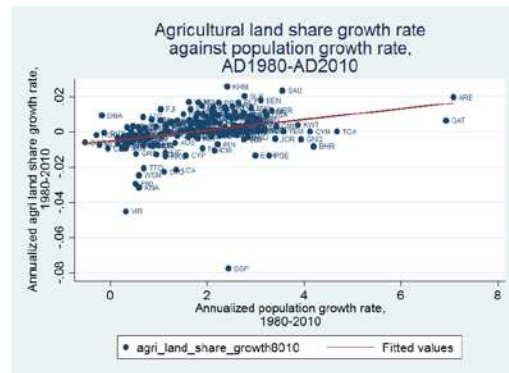


FIGURE 12B



Note: (Left) Annualized agricultural labor share growth rate against annualized population growth rate, from 37 countries where data are available in AD1980 and AD2010 (Note the outliers with annualized agricultural labor share growth rate ≤ -0.2 are excluded). (Right) Annualized agricultural land share growth rate against annualized population growth rate, from 207 countries where data are available in AD1980 and AD2010. The red line represents the fitted simple regression line. Source: World Bank (2016), employment in agriculture (% of total employment), agricultural land (% of land area) and population growth (annual %).